EE566 Solid State Devices

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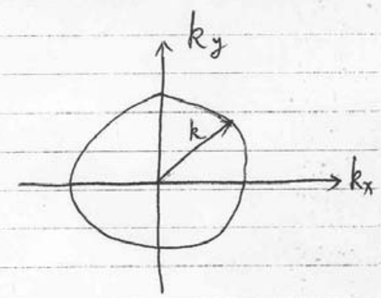
Assignment 2
SOLUTIONS

EE 566 - ASSIGNMENT 2 Solutions
Fall 2005 Solid State Devices 20/20

1. Assume the side of the 2D semiconductor is L
Then the density of states for 2D is

Liang →

$$g(E) = N(E)/E/L^2$$



$$N(E) = 2 \cdot \frac{1}{4} \cdot \frac{\pi^2 k^2}{(\pi/L)^2}$$

$$= \frac{1}{2} \frac{k^2 L^2}{\pi}$$

$$E = \frac{p^2}{2m^*} = \frac{\hbar^2 k^2}{2m^*} \Rightarrow k^2 = \frac{2m^* E}{\hbar^2}$$

$$\therefore N(E) = \frac{m^* E L^2}{\pi \hbar^2}$$

$$\therefore g(E) = \frac{m^*}{\pi \hbar^2}$$

$$n_{2d} = \int_{E_c}^{\infty} g(E) f(E)\, dE$$

$$= \int_{E_c}^{\infty} \frac{m^*}{\pi \hbar^2} \cdot \frac{1}{1+e^{(E-E_F)/kT}}\, dE$$

$$= \frac{m^*}{\pi \hbar^2} kT \int_{\frac{E_c - E_F}{kT}}^{\infty} \frac{1}{1+e^{(E-E_F)/kT}}\, d\left(\frac{E-E_F}{kT}\right)$$

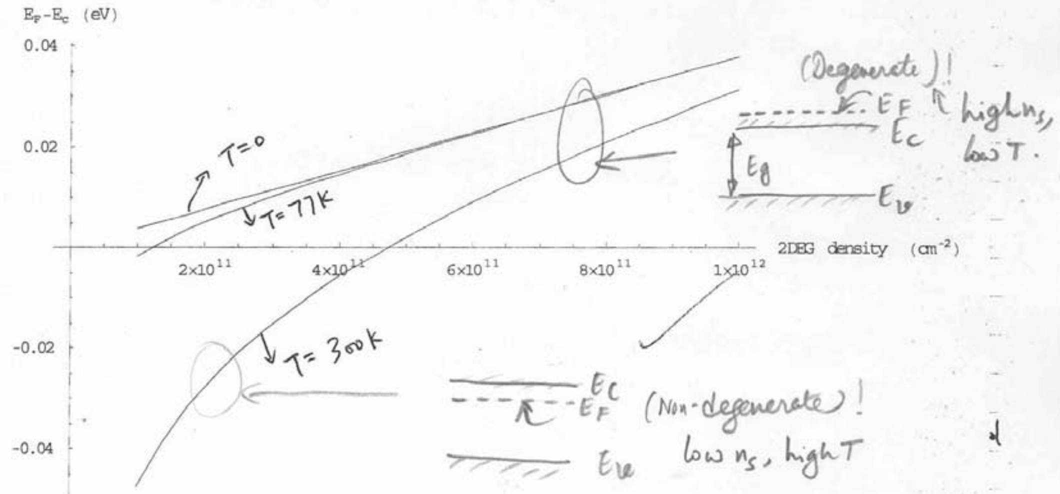
$$= \frac{m^*}{\pi \hbar^2} kT \cdot \ln\left(1 + e^{\frac{E_F - E_c}{kT}}\right)$$

Let $N_c^{2d} = \frac{m^*}{\pi \hbar^2} kT$, $\eta = \frac{E_F - E_c}{kT}$

Then $n_{2d} = N_c^{2d} (1 + e^{\eta})$

At $T=77K, 300K: E_F - E_c = kT \ln\left[\exp\left(\frac{n_{2d}\,\pi\hbar^2}{m^* kT}\right) - 1\right]$

At $T=0$, $E_F - E_c = \frac{n_{2d}\,\pi\hbar^2}{m^*}$



$$n_{2d} = \frac{m^* kT}{\pi\hbar^2} \ln\left(1 + e^{\frac{E_F - E_c}{kT}}\right)$$

From the above figure, it is shown that at low temperature the 2DEG carrier distribution is DEGENERATE.

$\therefore$ $E_F > E_c$ and for small T, $\frac{E_F - E_c}{kT} \gg 1$

then $n_{2d} = \frac{m^* kT}{\pi\hbar^2} \cdot \ln\left(e^{\frac{E_F - E_c}{kT}}\right) = \frac{m^*}{\pi\hbar^2}(E_F - E_c)$

$\therefore$ at low temperature, n_{2d} is independent of temperature.

6/6

So plot figure as follows. (fig. 1).

6/6 2DEG carrier distribution is degenerate

2 a) At room temperature, assume $n = N_D(x)$

When equilibrium condition is satisfied. $\vec{J} = 0$

i.e. $\vec{J} = q n \mu \vec{\mathcal{E}} + q D_n \frac{dn}{dx} = 0$

$$\Rightarrow \mathcal{E}(x,T) = \frac{-D_n}{\mu n} \frac{dn}{dx}$$
$$= \frac{-D_n}{\mu N_D(x)} \cdot \frac{dN_D(x)}{dx}$$

Solution

Jiang →

From $\frac{D_n}{\mu} = \frac{kT}{q}$

We get the relationship between $\mathcal{E}$ and T is

$$\mathcal{E}(x,T) = -\frac{kT}{q N_D(x)} \frac{dN_D(x)}{dx}$$

b) As temperature increase, the magnitue of $\mathcal{E}$ increases proportional to the temperature. The direction of $\mathcal{E}$ will not change. All we can say is that $\mathcal{E}\uparrow$ as $T\uparrow$.

$\mathcal{E} = f(N_D, n)$ which change with temperature too.

c) For constant doping $\frac{dN_D(x)}{dx} = 0$. $\Rightarrow \mathcal{E}(x,T) = 0$.

d)
$$\mathcal{E} = -\frac{kT}{q N_0 e^{-(\frac{x}{\lambda})^2}} \cdot N_0 e^{-(\frac{x}{\lambda})^2} \left(-\frac{2x}{\lambda^2}\right)$$
$$= +\frac{2kTx}{q\lambda^2}$$

Please refer to fig. 2.

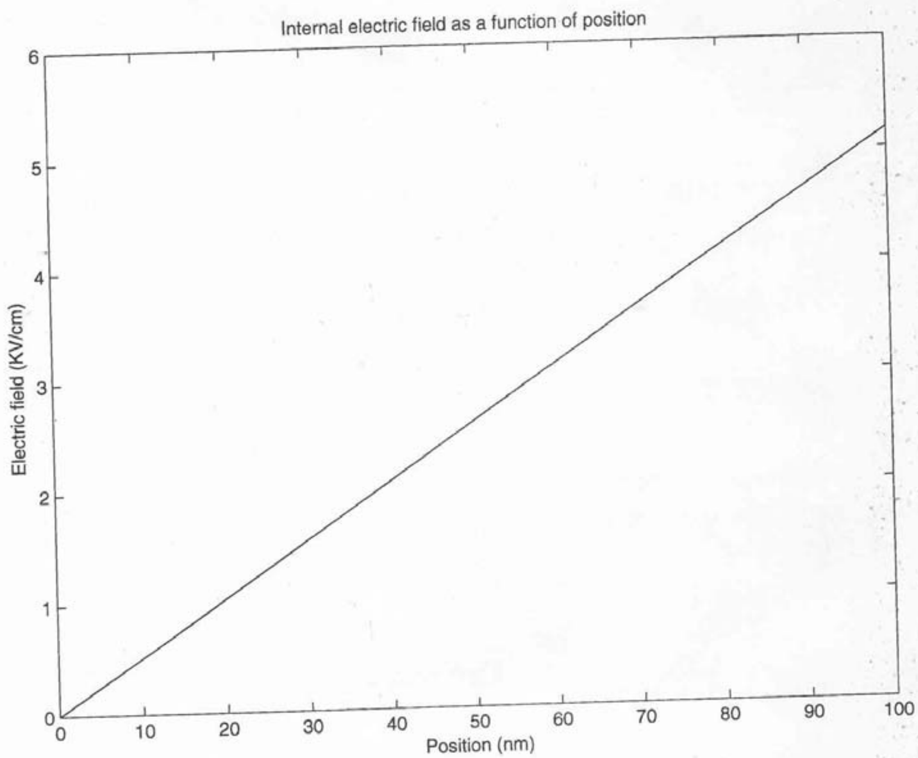
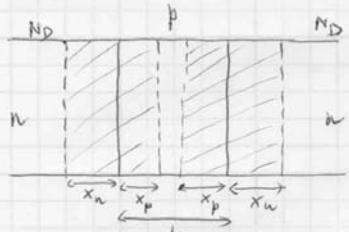

Internal electric field as a function of position
Electric field (KV/cm)
Position (nm)
0
1
2
3
4
5
6
0
10
20
30
40
50
60
70
80
90
100

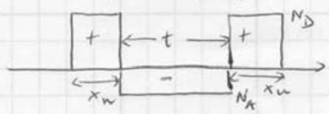
Fig 2.

SOLUTIONS ASSIGNMENT 2 - EE 566 - SOLID STATE DEVICES

Problem 3



↓ if p-layer totally depleted



The simplest design will be to shoot for only ONE capacitor - this is achieved if the p-region is totally depleted!

∴ Requirement : $\boxed{2x_p \geq t}$ — ①

Charge neutrality →

$$2x_n N_D = N_A t$$

$$\boxed{x_n = \frac{1}{2}\frac{N_A}{N_D}\cdot t} \quad — ②$$

Solution (He) → If p-region doping is N_A, the depletion width W of a 'single' p-n junction is given by -

$$\boxed{V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) = \frac{q}{2\epsilon_s}\left(\frac{N_A N_D}{N_A+N_D}\right)W^2} \quad — ③$$

= 0.84 Volt

From ① + ②, $(x_n + x_p)^2 = \boxed{W^2 = \left(1+\frac{N_A}{N_D}\right)^2 \frac{t^2}{4}}$ — ④

Combining ③ + ④, we get, by eliminating W, & using $d = 2x_n + t = t\left(1+\frac{N_A}{N_D}\right)$,

$$\boxed{\frac{1}{N_A} + \frac{1}{N_D} > \frac{q^2 d^2}{8\epsilon_s kT \ln\left(\frac{N_A N_D}{n_i^2}\right)}} \quad — ⑤$$

∴ $N_A < N_A^{cr}$, where N_A^{cr} is the solution of ⑤.

For $N_D = 10^{17}/cm^3$ + ~~N_A~~ $d = 300$ nm, I get

$N_A \leq 9.3 \times 10^{16}/cm^3$ & 't' scales accordingly.

For exactly $C = \frac{\epsilon_s}{300nm}$ @ $V_a = 0$, $N_A = 9.3\times10^{16}/cm^3$ & $t = 2x_p^{cr} = 2 * \frac{W}{\left(1+\frac{N_A}{N_D}\right)} = 2*(78nm)$ (W ← 150nm)

$\Rightarrow \boxed{t = 156\ nm}$ will work!

Check: $x_p = \frac{W}{1+\frac{N_A}{N_D}} = 78nm$ $x_n = 72nm \Rightarrow$ $W = 2(x_n + x_p) = 2x_n + t = 300nm$ ✓