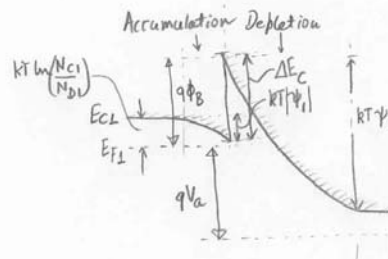
ASSIGNMENT 7 - SOLUTIONS

EE 566 - SOLID STATE DEVICES

Soln. by Debdeep Jena

PROBLEM 1

$$\boxed{J = J_0\, e^{\frac{-q\phi_B}{kT}} \left(1 - e^{-\frac{qV_a}{kT}}\right)} \quad (a)$$



CONDUCTION BAND - DIAGRAM

Conservation of energy →

$$\boxed{qV_a + q\phi_B = kT\psi_2 + kT\ln\left(\frac{N_{C2}}{N_{D2}}\right)} \quad (1)$$

Also,

$$\boxed{kT\ln\left(\frac{N_{C1}}{N_{D1}}\right) - kT|\psi_1| + \Delta E_C = q\phi_B} \quad (2)$$

Note:

$kT\psi_2$ is +ve

$kT\psi_1$ is negative, rewrite as $-kT|\psi_1|$ magnitude.

Combining (1) & (2), get

$$\boxed{\psi_2 + |\psi_1| = \frac{\Delta E_C}{kT} + \frac{qV_a}{kT} + \ln\left(\frac{N_{C1}}{N_{C2}}\frac{N_{D2}}{N_{D1}}\right)} \quad (3)$$

From continuity of electric displacement across junction,

$$\epsilon_{S1} F_1(0) = \epsilon_{S2} F_2(0)$$

also, we know that for band bending of $kT\cdot\psi$, and UNIFORM doping,

$$F(\psi) = \sqrt{\frac{2kTN_D}{\epsilon_S}\left(e^{\psi} + \psi - 1\right)} \cdot \quad \text{(Pg 23, Handouts)}$$

Applying this, get

$$\boxed{\psi_2 + |\psi_1| = \alpha e^{|\psi_1|} + (1-\alpha)(1 + |\psi_1|)} \quad (4)$$

From (3) & (4), & assuming $\frac{N_{C1}N_{D2}}{N_{C2}N_{D1}} \approx \frac{N_{D2}}{N_{D1}} \approx \frac{1}{\alpha}$,

$$\boxed{\alpha e^{|\psi_1|} + (1-\alpha)(1 + |\psi_1|) = \frac{\Delta E_C}{kT} + \frac{qV_a}{kT} - \ln\alpha} \quad (5)$$

From this point, the procedure is simple → Choose a V_a, solve (5) to get $|\psi_1|$, & use (2) to get corresponding $q\phi_B$, & use (a) to get J. Repeat for all V_a values, for fixed α for any given structure.

Problem 1 continued...

In general, one needs to solve the transcedental eqn 5. However, for the specific case of $\alpha = 1$ (same doping on both sides), 5 becomes

$$e^{|\psi_1|} = \frac{\Delta E_C + qV_a}{kT}$$

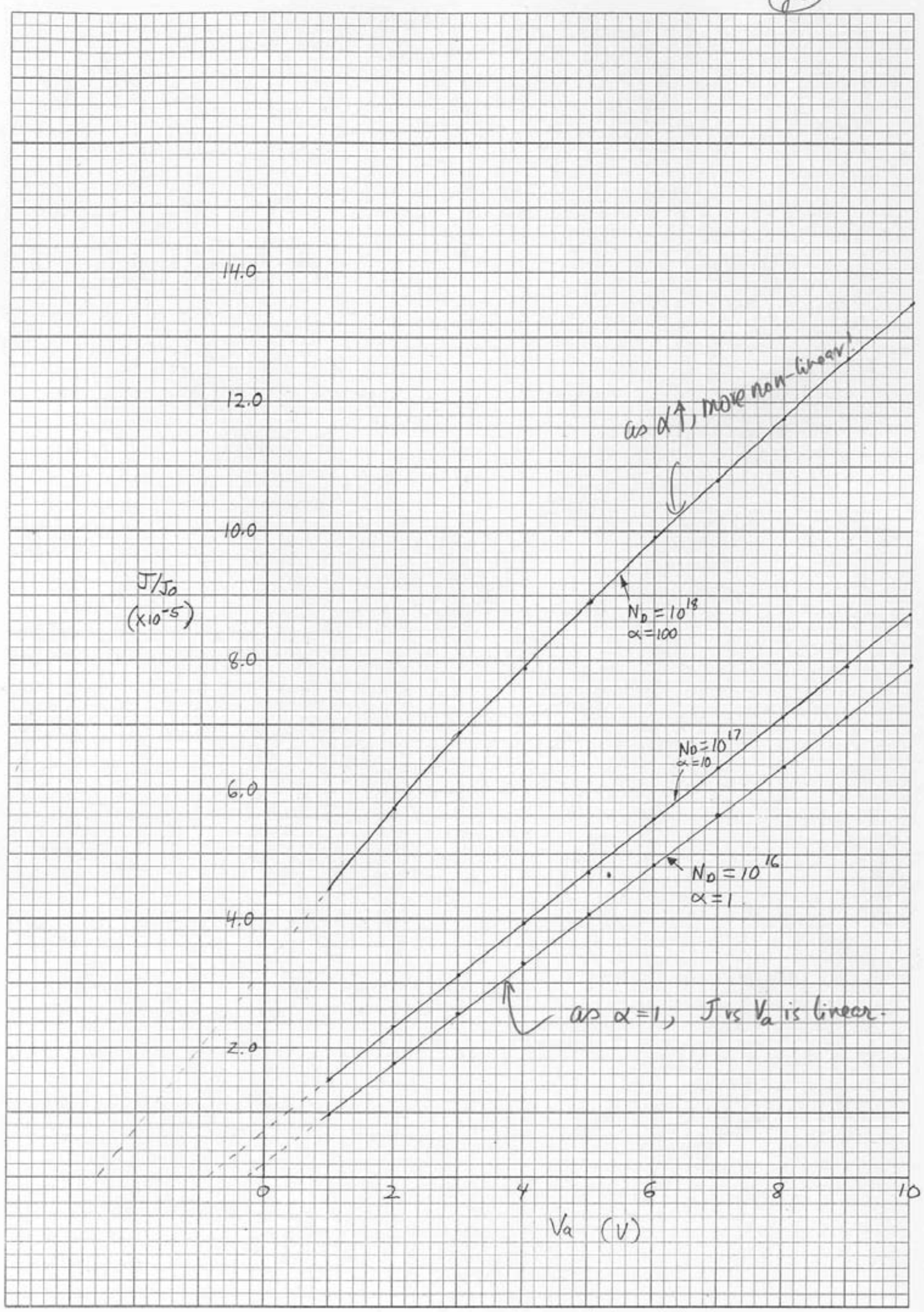
Under strong accumulation ($\alpha \to 1$, N_{D1} low, $e^{|\psi_1|} \gg |\psi_1|$),

$$J/J_0 \propto V_a \quad \leftarrow \text{linear relationship.}$$

$\Leftarrow$ BEHAVES LIKE A OHMIC CONTACT!

As α gets larger than 1, J/J_0 starts becoming non-linear.

See the calculated plots (by Greg Snider) in the next page.

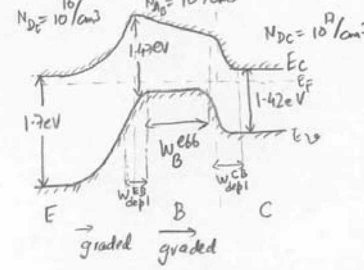


10 Squares to the Inch

Calculated by G. Snider.

PROBLEM 2 — HBT

(a)



$$\underbrace{W_B^{eff}}_{\text{Effective Quasi-neutral base width.}} = W_B^{total} - \underbrace{\left(x_p^{EB} + x_p^{BC}\right)}_{\text{Depletion widths of base layer}}$$

Crank through old formulae to get

$$x_p^{BC} = \frac{N_{DC}}{N_{AB}+N_{DC}} W_{depl}^{CB} \leftarrow \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_{DC}} + \frac{1}{N_{AB}}\right)\left(V_{bi} - 2\frac{kT}{q}\right)}$$

$$\approx 13\,nm$$

Similarly, $x_p^{EB} \approx 4\,nm$ $\left(= \frac{N_{DE}}{N_{DE}+N_{AB}} W_{depl}^{EB} \right)$

$\Rightarrow$ $W_B^{eff} = 200 - (4+13) = 187\,nm$.

Bandgap change over $W_B^{eff} = \Delta E_g^{base} * \left(\frac{187nm}{200nm}\right) \approx \Delta E_g^{base}$

Now since base is doped p-type, E_v is flat, $\Delta E_C = \Delta E_g$.

$\Rightarrow$ Quasi-Electric field in the base is

$$\boxed{|F_{QE}| = \frac{\Delta E_g^{base}}{q W_B^{eff}} \approx \frac{50\,meV}{187\,nm} \approx 2.7\,kV/cm}$$

See my 1D Poisson simulations attached in the next page.

(b)

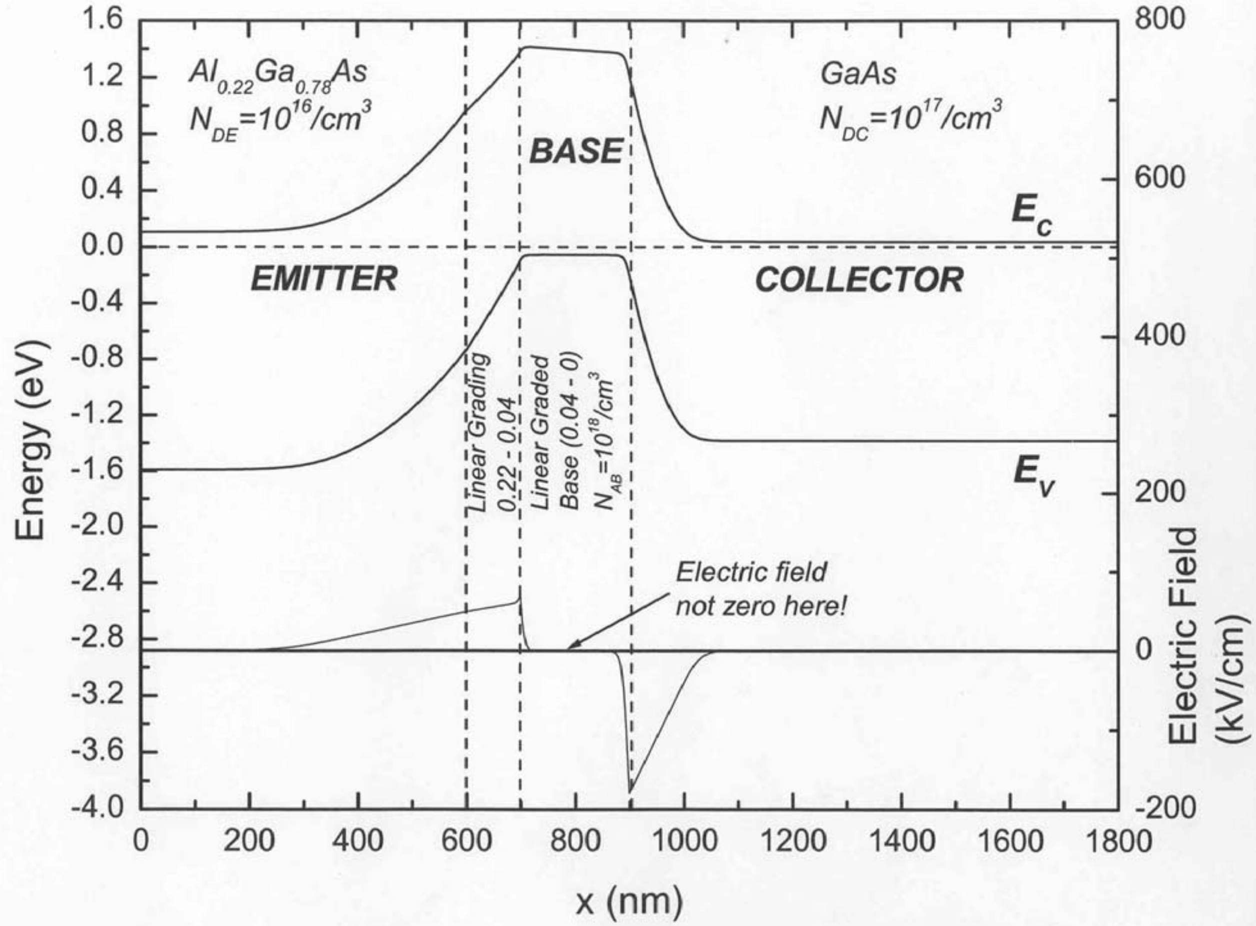
$$\boxed{n_B(x) = \frac{J_C}{q\mu_n |F_{QE}|}\left[1 - \exp\left[-\frac{q|F_B|}{kT}(W_B^{eff} - x)\right]\right]} \quad \text{— (4a)}$$

I get this from the drift-diffusion equation by writing

$$J_C = q\, n(x)\, \mu_n |F_{QE}| - q D_n \frac{dn(x)}{dx}$$ — linear ordinary differential equation.

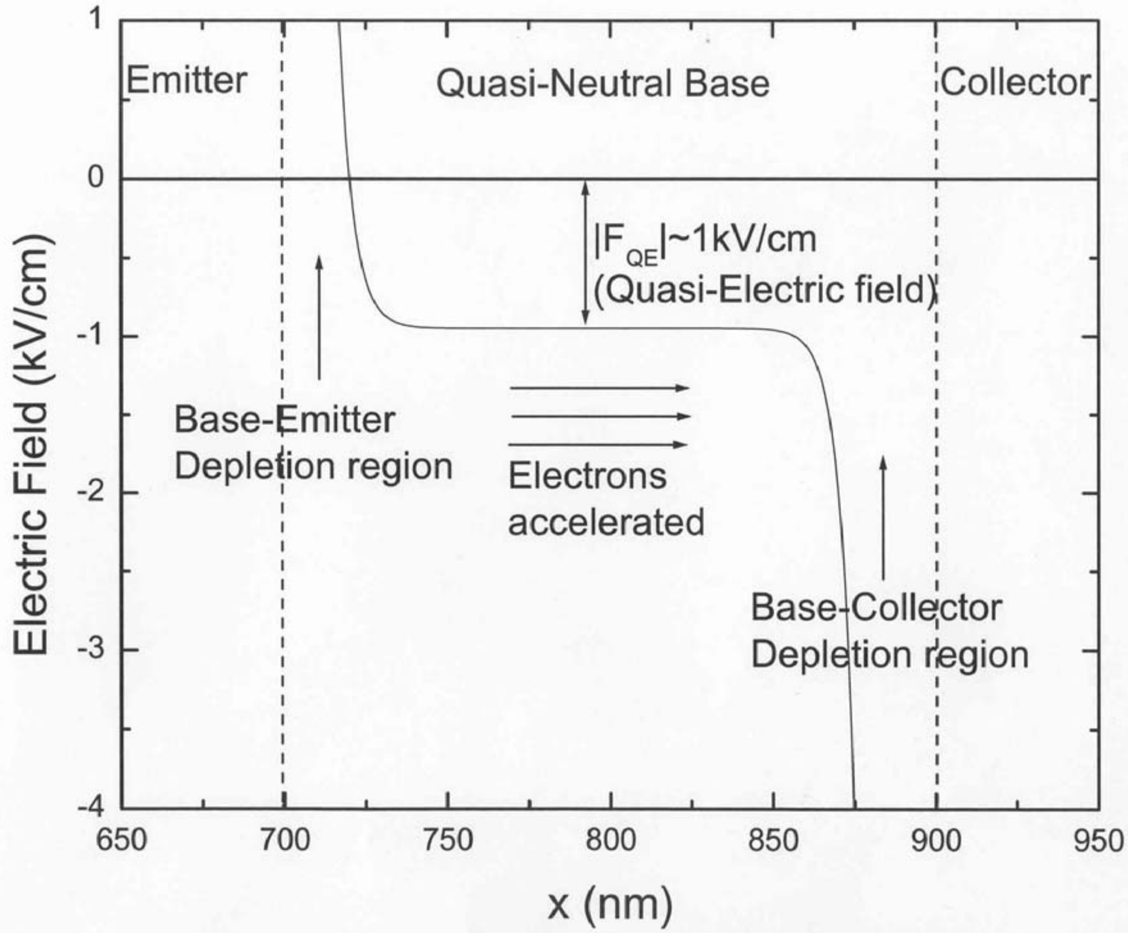
Solve this with the 2 boundary conditions

$$\left. \begin{matrix} n_B(0) = n_{p0}\, e^{\frac{qV_{BE}}{kT}} \quad \& \quad n_B(W_B^{eff}) = 0 \end{matrix} \right\}$$ & get (4a) easily.



Al0.22Ga0.78As
NDE=10^16/cm^3
GaAs
NDC=10^17/cm^3
BASE
EMITTER
COLLECTOR
Ec
Ev
Linear Grading
0.22 - 0.04
Linear Graded
Base (0.04 - 0)
NAB=10^18/cm^3
Electric field
not zero here!
Energy (eV)
Electric Field
(kV/cm)
x (nm)
1.6
1.2
0.8
0.4
0.0
-0.4
-0.8
-1.2
-1.6
-2.0
-2.4
-2.8
-3.2
-3.6
-4.0
800
600
400
200
0
-200
0
200
400
600
800
1000
1200
1400
1600
1800

5a


Emitter
Quasi-Neutral Base
Collector
|F_QE|~1kV/cm
(Quasi-Electric field)
Base-Emitter
Depletion region
Electrons
accelerated
Base-Collector
Depletion region
Electric Field (kV/cm)
x (nm)
1
0
-1
-2
-3
-4
650
700
750
800
850
900
950

(5b)

(c) For analytical expression, see Kroemer (Solid State Electronics, vol 88, pg 1101, 1985) (+ handout, pg 77)

$$\tau_{tr}(\text{graded}) \approx \underbrace{\tau_{tr}(\text{ungraded})}_{(W_B^{eff})^2/2\tilde{D}_n \;\approx 0.8\text{ps}} * \underbrace{\frac{2kT}{\Delta E_g}}_{\frac{2*(52\text{ meV})}{50\text{ meV}}}$$ ← holds only for $\Delta E_g >> kT$

50 meV ← ΔE_g is not $>> kT$.

More accurate form is

$$\tau_{tr}(\text{graded}) \approx \frac{W_B^{eff}}{\mu_n |F_{BE}|}\left\{1 - \frac{kT}{q|F_{BE}|W_B^{eff}|}\right\}$$

For our case, plug in all values to get

$$\tau_{tr}(\text{graded}) \approx 0.4\text{ps}$$

$$\boxed{\frac{\tau_{tr}(\text{graded})}{\tau_{tr}(\text{ungraded})} = \frac{0.4\text{ps}}{0.8\text{ps}} \approx \frac{1}{2}}$$

(d)

$$\gamma_E = \frac{1}{1 + \frac{GN_B}{GN_E}\frac{\tilde{D}_{pE}}{\tilde{D}_{nB}} \cdot \exp\left(-\frac{\Delta E_g}{kT}\right)} = 0.9992$$

(use effective W_E^{eff} for GN_E + W_B^{eff} for GN_B)

for $\underline{V_{BE} = 0.8\text{ Volt}}$.

$$\alpha_T = 1 - \frac{\tau_{tr}}{\tau_n} = 0.999953$$

$V_{BE} = 0.8$ volt → get $\beta_F(\text{graded}) \sim 1200$

$\beta_F(\text{ungraded}) \sim 1000$

Not much improvement in β_F due to base grading.

(e) Base grading helps make the HBT faster by reducing base transit time.

A homojunction BJT with similar doping densities would have ~~the~~

$$\beta_F \propto \frac{GN_E}{GN_B} < 1$$, since base is doped more heavily than emitter!!

On the other hand, by doping base heavily, one can make HBT faster since $R_B \downarrow$.

So HBT gives much more freedom in design.