EE 566
SPR 2004
02/02/04

SOLID STATE DEVICES

ASSGN II - SOLNS.

(1)

a)

$$J_n(x) = q\, n(x)\, \mu_n F(x) + q D_n \frac{dn(x)}{dx} = 0$$

$$\therefore \quad F(x) = -\frac{D_n}{\mu_n} \frac{dn(x)/n(x)}{dx}$$

$$= -\frac{D_n}{\mu_n} \frac{\partial \ln(n(x)/N_C)}{\partial x}$$

b)

$$n(x) = N_C \exp\left(\frac{E_F - E_C(x)}{kT}\right)$$ for non-degenerate semiconductor.

$$\frac{\partial \ln(n(x)/N_C)}{\partial x} = -\frac{1}{kT}\frac{dE_C(x)}{dx} = -\frac{q}{kT} F(x) \quad \left(\text{Since } F(x) = \frac{1}{q}\frac{dE_C(x)}{dx}\right)$$

$\Rightarrow$ from (a),

$$F(x) = \frac{-D_n}{\mu_n} \cdot \left(-\frac{q}{kT}\right) \cdot F(x)$$

$$\Rightarrow \quad \boxed{\frac{D_n}{\mu_n} = \frac{kT}{q}}$$

$$D_n = \mu_n \cdot \frac{kT}{q} = \left(1000 \frac{cm^2}{V \cdot s}\right) * (26\, mV) = \boxed{26 \frac{cm^2}{s}}$$

Typical values of D_n's are of this order.

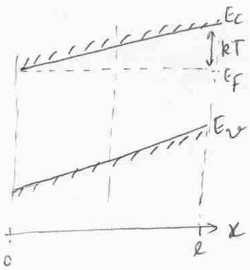
c)

$$n(x) = N_C \exp(-x/\ell)$$

Note that this is actually degenerate, as some of you have pointed out ... However, we solve it in the non-degenerate limit to illustrate a couple of points ...

c) contd... $@ \; x = 0 \Rightarrow n = N_C \Rightarrow E_F = E_C$.

$x = l \Rightarrow n = N_c \exp(-1) \Rightarrow E_F = E_C - kT$



$$J_n^{drift}(x = l/2) = q\, n(l/2)\, \mu_n\, F(l/2)$$

$$F(x) = \frac{-D_n}{\mu_n} \frac{\partial \ln(n(x)/N_c)}{\partial x} = \frac{D_n}{\mu_n l}$$

$$\therefore \quad J_n^{drift} = q N_c \exp(-\tfrac{1}{2}) \cancel{\mu_n} \frac{D_n}{\cancel{\mu_n} l} = q \frac{N_c D_n}{l} \sqrt{e}$$

$$J_n^{diff}(x = l/2) = q D_n \frac{dn(x)}{dx}\Big|_{x = l/2} = \underset{\uparrow \; -ve}{-}\, q \frac{D_n}{l} N_c \sqrt{e}$$

$\therefore \; J_n^{drift} + J_n^{diff} = 0$, as it should be!

d) Since $n(x) = N_c F_{\frac{1}{2}}\Big(\overbrace{\frac{E_F - E_c(x)}{kT}}^{\eta} \Big)$, & $\left\{ \begin{array}{l} F(x) = \frac{1}{q} \frac{dE_c(x)}{dx}, \\ \frac{dF_j(\eta)}{d\eta} = F_{j-1}(\eta) \end{array} \right.$

$$\frac{1}{n(x)} \frac{dn(x)}{dx} = \frac{F_{-\frac{1}{2}}(\eta)}{F_{+\frac{1}{2}}(\eta)} \cdot \left(-\frac{q}{kT} F(x) \right)$$

$\Rightarrow$ from a), $F(x) = \frac{D_n}{\mu_n} \cdot \frac{F_{-\frac{1}{2}}(\eta)}{F_{+\frac{1}{2}}(\eta)} \cdot \frac{q}{kT} \cdot F(x)$

$$\Rightarrow \boxed{\frac{D_n}{\mu_n} = \frac{kT}{q} \cdot \frac{F_{\frac{1}{2}}(\eta)}{F_{-\frac{1}{2}}(\eta)}}$$

for non-degenerate case, $F_j(\eta) \approx \exp(\eta)$, & $\frac{D_n}{\mu_n} = \frac{kT}{q}$.

e) Degenerate semiconductor:

$$F_{\frac{1}{2}}(\eta) \approx \frac{\eta^{3/2}}{\Gamma(5/2)} = \frac{4}{3\sqrt{\pi}} \frac{(E_F - E_C)^{3/2}}{(kT)^{3/2}}$$

So, CB electron density is

$$n = N_C \, F_{1/2}(\eta) \approx 2\left(\frac{m^* \cancel{kT}}{2\pi\hbar^2}\right)^{3/2} * \frac{4}{3\sqrt{\pi}} \frac{(E_F - E_C)^{3/2}}{\cancel{(kT)^{3/2}}}$$

$$= \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2} \cdot \underbrace{\left[\frac{2}{3}(\varepsilon_F - E_C)^{3/2}\right]}_{= \int_0^{E_F} d\varepsilon \, (\varepsilon - \varepsilon_c)^{1/2}}$$

$$= \int_0^{E_F \leftarrow \text{Fermi level}} d\varepsilon \, g_{3D}(\varepsilon)$$

↑ 3D density of states ← $g_{3D}(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon - \varepsilon_c}$

$\therefore \quad n \sim (\varepsilon_F - \varepsilon_c)^{3/2}$, it has <u>no explicit T-dependence</u>.

Non-degenerate carriers have a strong T-dependence, since the carriers are thermally activated by energy kT.

Degenerate carriers are "activated" by their "degeneracy", i.e., by Pauli-Exclusion, and the energy is $(\varepsilon_F - \varepsilon_c)$.

Thus, all transport properties of degenerate carriers may be approximated by replacing 'kT' of the property by $(\varepsilon_F - \varepsilon_c)$!

For example, $\boxed{\frac{D_n}{\mu_n} = \frac{2/3(E_F - E_C)}{q}}$ for degenerate SC's!