EE566 Solid State Devices

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Assignment 2
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Einstein Relation and Degenerate semiconductors

You will derive the Einstein relation from physical arguments on current flow. Consider a bulk semiconductor with a carrier density profile *n(x)* and no *external* electric field.

a) Show that the condition of zero net current flow (drift + diffusion) implies that an *internal* electric field is developed to oppose current flow, and is given by the most general case by

$$F(x) = -\frac{D_n}{\mu_n} \cdot \frac{\partial \ln[n(x)/N_C]}{\partial x},$$

where N_C is the conduction band effective density of states, μ_n is electron mobility, and D_n is the electron diffusion constant. Remember that the condition of no current flow implies that the Fermi level is constant, i.e. $dE_F(x)/dx=0$.

b) Now assume a *non-degenerate* semiconductor ($E_C - E_F > 3kT$). Using the result from part a, show that

$$\frac{D_n}{\mu_n} = \frac{kT}{e}.$$

This is the Einstein relation for non-degenerate semiconductors derived in class from different considerations. Calculate the value of the diffusion constant at room temperature for a mobility of 1000 cm^2/V.s. *Remember* the order of magnitude.

c) Find an expression for, and *sketch* the band-diagram ($E_C(x)$, $E_V(x)$, $E_F(x)$, for $0<x<l$) for a non-degenerate carrier profile

$$n(x) = N_C \exp(-x/l).$$

Find the drift and diffusion currents at $x=l/2$. What is the *total* current at $x=l/2$?

d) (Hard!) Show that the *general* Einstein relation is given by

$$\frac{D_n}{\mu_n} = \frac{kT}{e} \cdot \frac{F_{1/2}(\eta)}{F_{-1/2}(\eta)},$$

where $\eta = (E_F - E_C)/kT$, and $F_j(x)$ is the j^{th} order Fermi-Dirac integral. You might need the following identity - $dF_j(\eta)/d\eta = F_{j-1}(\eta)$ and the Gamma functions $\Gamma(1/2) = \sqrt{\pi}$, and $\Gamma(j+1) = j\Gamma(j)$. Show that this result reduces to the simple form in part (b) for a non-degenerate semiconductor.

e) Now suppose the semiconductor is heavily doped such that it is degenerate. What is the dependence of conduction band electron density on temperature for a degenerate semiconductor? You will need the approximation $F_j(\eta) \approx \eta^{j+1}/\Gamma(j+2)$ for degenerate semiconductors ($E_F - E_C > 3kT$). Finally, what is the Einstein relation for this density? How is it different from a non-degenerate semiconductor? Remember this general rule – for most cases, a corresponding expression for a degenerate semiconductor property can be got by the replacement $kT \to (E_F - E_C)$ in the non-degenerate expression. Can you give a physical basis for this?