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ASSIGNMENT 10 - SOLUTIONS
EE 566 - SOLID STATE DEVICES

Soln. by Dabdeep Jena

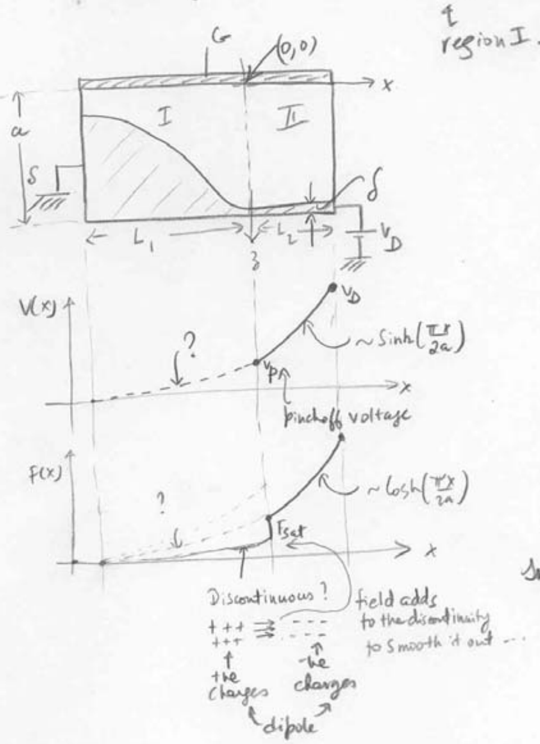
PROBLEM I

MESFET 2-D problem:

The problem was assigned to ensure you read the paper. The main difficulty with the 2-region model is matching the Field $\left(\frac{\partial V}{\partial x}\Big|_{(0,a^-)} = \frac{\partial V}{\partial x}\Big|_{(0,a^+)}\right)$

↑ region I. ↖ Region 2, $= F_{sat}$

⇓



A simple-minded solution by calculating $V(x, z=a)$. with $\begin{cases} V(x=-L_1, z=a) = 0 \ \& \\ V(x=0, z=a) = V_p = \frac{qN_d a^2}{2\epsilon_s} \end{cases}$

fails to match the electric fields. (Believe me, I did this myself too ☺).

⇓

If there is a field discontinuity, it can create a charge-dipole to smoothen it out!

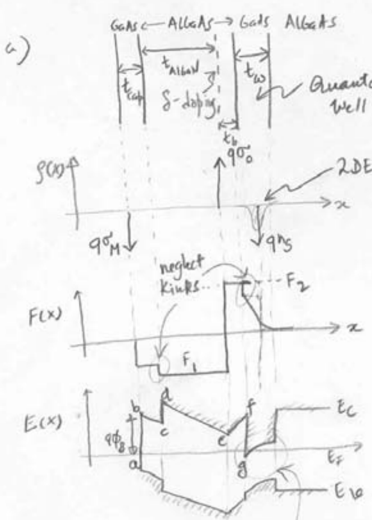
Check this out in pg 129c figs d & f ...

PROBLEM 2 HEMT design

(2)

We need $n_S = 10^{12}/cm^2$, what's the reqd. sheet doping?

a)



Assume sheet doping $= q\sigma_0$ coul/cm²

Charge conservation: $q\sigma_M + qn_S = q\sigma_0$

Go around the loop in the band diagram

$E_F \to a \to b \to c \to d \to e \to f \to g \to E_F$

$+ q\phi_B - F_1 * t_{cap} + \Delta E_C - F_1 * t_{AlGaN} + F_2 * t_b - \Delta E_C + (\varepsilon_0 - E_c) + (E_F - \varepsilon_0) = 0$

$F_1 = \dfrac{q\sigma_M}{\epsilon_S}$, $F_1 = \dfrac{q\sigma_M}{\epsilon_S}$, $F_2 = \dfrac{qn_S}{\epsilon_S}$ (a) (b)

(a) $\boxed{\varepsilon_0 - \varepsilon_c = \text{Ground-state of a q-well.}}$

$\left\{\begin{array}{l}\text{assume } \infty\text{-well} \\ t_\omega = 15nm \\ \uparrow \\ \text{Can assume } \Delta \text{well too...}\end{array}\right\} \approx 25 meV$ (See pg 33 of NOTES)

$2D\ DOS = \dfrac{m^*}{\pi\hbar^2}$

$(E_F - \varepsilon_0) * g_{2D}(\varepsilon) \approx n_S$

$\therefore$ (b) $\boxed{(E_F - \varepsilon_0) \approx \dfrac{\pi\hbar^2 n_S}{m^*}}$

also, $q\sigma_M = q(\sigma_0 - n_S)$

$\Rightarrow$

$$q\phi_B - \frac{q(\sigma_0 - n_S)}{\epsilon_S}(t_{cap} + t_{AlGaN}) + \frac{qn_S}{\epsilon_S} t_b + \underbrace{(\varepsilon_0 - \varepsilon_c)}_{25meV}/q + \underbrace{(E_F - \varepsilon_0)}_{\frac{\pi\hbar^2}{m^*} n_S}/q = 0$$

Effective Bohr Radius

$$\Rightarrow \boxed{\sigma_0 \overset{\text{approx...}}{\approx} \frac{\epsilon_S}{q(t_{cap} + t_{AlGaN})}\left[\phi_B + \frac{\varepsilon_0 - \varepsilon_c}{q}\right] + n_S * \left\{1 + \frac{t_B + \pi \frac{\hbar^2 \epsilon_S}{q^2 m^*}}{t_{cap} + t_{AlGaN}}\right\}}$$

$\approx 2.1 * 10^{12} + 1.9 * 10^{12}$

$\boxed{\sigma_0 \approx 4 * 10^{12}/cm^2}$ ⇐ Note: This is a crude estimate. If you assume a Δ-Quantum well, you'll get $\sigma_0 \approx 3.3 * 10^{12}/cm^2$.

The method is more important here.

Prob 2 : Contd...

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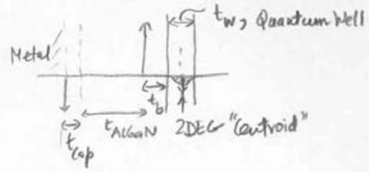
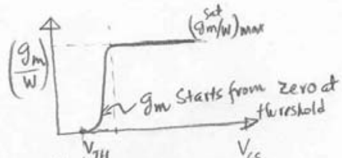
(b)

$$\left(\frac{g_m^{Sat}}{W}\right)_{max} = C_g v_{Sat} \approx \left(\frac{\epsilon_s}{t_{M \to 2DEG}}\right) * (v_{Sat})$$ ← Short-channel transconductance...



approximate

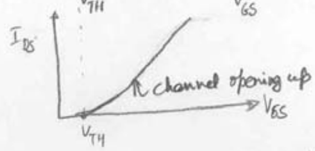
$$t_{M \to 2DEG} = t_{Cap} + t_{AlGaN} + t_b + t_w/2$$



$$\left(\frac{g_m^{Sat}}{W}\right)_{max} \approx \frac{13*8.85*10^{-14}*10^{7}}{(5+30+2+7.5)*10^{-7}} \frac{S}{cm} = 2.58\ S/cm$$

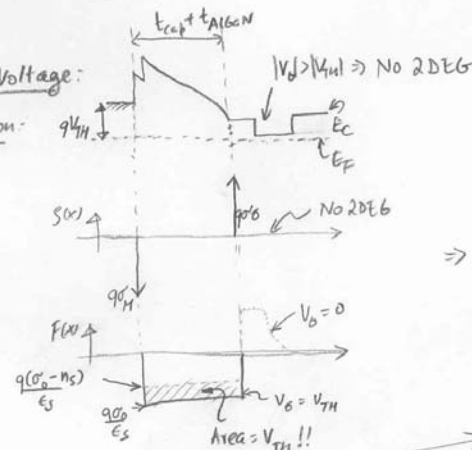
$$= 258\ S/m$$

$$\boxed{\left(\frac{g_m^{Sat}}{W}\right)_{max} \approx 258\ mS/mm}$$ Constant at saturation region.



Threshold Voltage:

Approximate solution:



$$\Rightarrow |V_{TH}| = \left\{\frac{q\sigma_0}{\epsilon_s} - \frac{q(\sigma_0 - n_s)}{\epsilon_s}\right\} * (t_{cap} + t_{AlGaN})$$

$$= \frac{q n_s}{\epsilon_s} * (t_{cap} + t_{AlGaN})$$

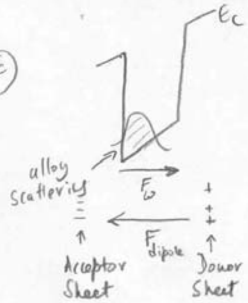
$$= 0.48 \text{ Volts.}$$

Note:

$$|V_{TH}| = \frac{q n_s \leftarrow \text{2DEG charge}}{\left[\epsilon_s / (t_{cap} + t_{AlGaN})\right] \leftarrow \text{capacitance}}$$

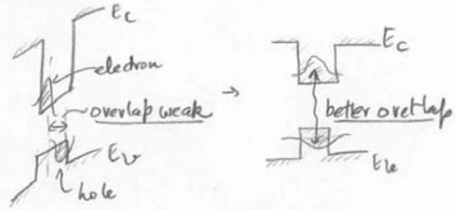
$\therefore \boxed{V_{TH} \approx -0.48\ \text{Volt}}$

(c)



One can dope an "inverse" sheet dipole to cancel the electric field in the well. It needs an equal & opposite donor-acceptor sheet (or δ-) doping as shown.

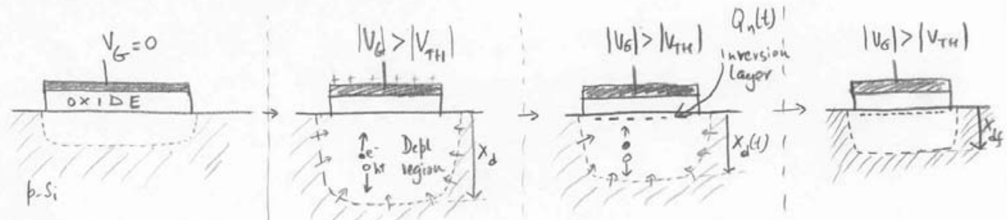
(d)



If there is an electric field in a QWell, & you want to use it for an optical device, the electron & holes are pushed in opposite directions & the chances of them recombining & giving out photons is reduced. This is called the Quantum-Confined Stark Effect (QCSE). So for good optical devices, it is highly desirable to have flat wells so that e-h overlap is high.

If I flatten the well for the HEMT though with doping, ionized impurity scattering will increase, & chances are that mobility will go down. However, alloy scattering, which occurs due to penetration of 2DEG wavef^n into the barrier is reduced, since the 2DEG is pushed away from the surface...
It's always give and take ☺

PROBLEM 3 MOS Capacitor (5)



$\boxed{t=-\infty}$ Steady State

$\boxed{t=0}$: Inversion charge does NOT form immediately

$\boxed{t=t}$: Inversion layer partially formed from thermally generated ehp^s in depl. region ($np < n_i^2$)

$\boxed{t=\infty}$: Inversion layer completely formed. $x_{depl} = x_{df}$ is max. depletion thickness @ eq^m.

A pictorial view of the mechanism (of DRAM op^n).

$$\frac{dQ_n}{dt} = -\frac{q n_i (x_d - x_{df})}{2\tau_0}$$ → use this, & simple charge conservation

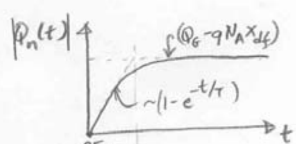
$$|Q_n(t)| + q N_A x_d(t) = |Q_G|$$

get, in, eliminate $x_d(t)$

(a) $$Q_n + \underbrace{\left(\frac{2\tau_0 N_A}{n_i}\right)}_{T} \frac{dQ_n}{dt} = -[Q_G - q N_A x_{df}]$$

(b) Solve: $$\int_{Q_n(0)=0}^{Q_n(t)} \frac{dQ_n}{(Q_n + Q_G - q N_A x_{df})} = -\int_{t=0}^{t} \frac{dt}{T}$$

$$\ln\left(\frac{Q_n(t) + Q_G - q N_A x_{df}}{Q_G - q N_A x_{df}}\right) = -t/T$$



No inv. charge at $t=0$

$$\Rightarrow \boxed{Q_n(t) = -(Q_G - q N_A x_{df})(1 - e^{-t/T})}$$

$$T = 2\tau_0 * \frac{N_A}{n_i} = (2*10^{-6}s)*\left(\frac{10^{15}/cm^3}{10^{10}/cm^3}\right) \approx \boxed{0.2 \text{ seconds}}$$

Since the time reqd' to form the layer is in the order of seconds, one can come back to the MOS capacitor after many clock cycles & find what had been stored there before ⇒ it is a memory...

PROBLEM 4 (Soln. by Qin Zhang, my comments ...) (6)

P4. a) $\uparrow y$ $\longrightarrow x$ (channel)

$$J_{drift}(x,y) = q\mu_n n(x,y) * F_x$$

$$I_{drift}(x) = \int_0^{y_i} J_{diff}(x,y)\, Z\, dy$$

$\int n(x,y)dy = \int n(x,y)\frac{dy}{d\psi}\cdot d\psi$

$= \int \frac{n(\psi)\,d\psi}{(\partial\psi/\partial y)}$ ← related to field

in linear $F_x \approx V_{DS}/L$ $$= \frac{q\mu_n V_{DS} Z}{L}\int_0^{y_i} n(x,y)\,dy$$

strong inversion $$= \frac{q\mu_n V_{DS} Z}{L}\int_{\psi_B}^{\psi_S} \frac{n_{po} e^{\frac{q}{kT}(\psi - V(x))}}{(\sqrt{2}kT/qL_D) F\left(\frac{q\psi}{kT}, V(x), n_{po}/p_{po}\right)}\, d\psi$$

$$\boxed{I_{drift}(x) = \frac{q^2\mu_n V_{DS} Z\, n_{po} L_D}{\sqrt{2}\, kT L}\int_{\psi_B}^{\psi_S} \frac{e^{\frac{q}{kT}(\psi - V(x))}}{F\left(\frac{q\psi}{kT}, V(x), n_{po}/p_{po}\right)}\, d\psi}$$

where $V(x) \approx \frac{V_{DS}}{L}\cdot x$.

$$F\left(\frac{q\psi}{kT}, V(x), n_{po}/p_{po}\right) = \left[e^{-\frac{q\psi}{kT}} + \frac{q\psi}{kT} - 1 + \frac{n_{po}}{p_{po}} e^{-\frac{qV(x)}{kT}}\left(e^{\frac{q\psi}{kT}} - \frac{q\psi}{kT} e^{\frac{qV(x)}{kT}} - 1\right)\right]^{1/2}$$

$$J_{diff}(x,y) = q D_n \nabla n$$

$$I_{diff}(x) = \int_0^{y_i} Z q D_n \frac{dn(x,y)}{dx}\, dy$$

$$= \frac{d}{dx}\int_0^{y_i} Z q D_n\, n(x,y)\, dy$$

$$= q D_n Z \frac{d}{dx}\int_{\psi_B}^{\psi_S} \frac{n_{po} e^{\frac{q(\psi - V(x))}{kT}}}{(\sqrt{2}kT/qL_D) F\left(\frac{q\psi}{kT}, V(x), n_{po}/p_{po}\right)}\, d\psi$$

$$\boxed{I_{diff}(x) = \frac{q\mu_n Z\, n_{po} L_D}{\sqrt{2}}\frac{d}{dx}\int_{\psi_B}^{\psi_S} \frac{e^{\frac{q}{kT}(\psi - V(x))}}{F\left(\frac{q\psi}{kT}, V(x), n_{po}/p_{po}\right)}\, d\psi}$$

b) J_{drift} dominates

(V_{DS} is small, $\frac{dn}{dx}$ is small)