Problem Set 6:

1. Suppose that there is a town of people living along a line from zero to 1. The closer you live to point 0, the more you like choir music at church, the closer you live to point 1 the less you like choir music. Your utility from going to church *c* is $U = e_c - |x - x_c|$. If you don't go to church your utility is zero. If you go to church, you'll go to the church that gives you the highest utility. People in the town are 'evenly spread out' so that half the people live between zero and ½, and so on.

Suppose there were 2 churches in this town. To make life easy let's assume that both churches have their effort levels set to 1. Also, let's suppose that there are 101 possible locations x_c that a church could choose to be out evenly spread out through the town. So x_c could be 0, 0.01, 0.02, all the way up to 1. Finally, let's suppose that each church wants as many members as possible.

A. Verify to yourself that if there was just one church in town with effort level 1, then everybody would go to church no matter where the church was located.

   It is straightforward to show that for any person whose distane from zero is less than 1 (that is, everyone in town) the utility from attending is at least as large as the utility from staying home.

B. Now let's think about 2 churches in town. Suppose the first church was located at point zero and church two was deciding where it would locate itself (that is how much choir music it would use). Where should church 2 go given that church 1 is at zero?

   The first thing to realize is that given that each church has the same effort level, people will go to the closest church (and from part A we can deduce that everyone is going to go to church, nobody stays home.) So with that in mind, church 2 would locate right next door to church 1—at point 0.01. Then everyone will go to church 2!

C. Now suppose church 1 was at ¼ and church 2 had to decide where to locate. Where would it locate in this case?

   Again it would go next door—at point 0.26. Now church 2 gets about ¾ of the town and church 1 only gets ¼!

D. Now suppose that church 1 was located at 0.49. Where would church 2 locate now?

   Now church 2 would go to 0.5, and each church would get about half the town.

E. In parts B, C, and D, would church 1 be happy about things once church 2 made it's decision? Or would church 1 want to change what it was doing in each case? What do you think the Nash Equilibrium here is?

   In parts B and C, church 1 would not be happy! It would want to move a bit to the right to turn the tables on church 2. In part D, church 1 cannot do any better—if it moves to point 0.51 it will

have the same fraction of members as before. So in part D neither church has an incentive to change, this is in fact a Nash Equilibrium.

F. The above problem is a famous one created to explain why similar stores (e.g., gas stations, hardware stores, etc) locate near each other in a town. But in our case spatial distance is a metaphor for religious content. Suppose there was a town with 2 churches. Based on this problem would expect these churches to have very similar content or very different religious content from each other?

From the above question, we would assume that these churches would be very similar to each other.

2. Suppose there is a town of people located along a line from zero to 1. Suppose that there are two churches in the town located at 0 and 1. As usual let the utility from going to church *c* be $U = e_c - |x - x_c|$. In class we assumed that each church maximized $f - \frac{1}{2}e^2$, where *f* is the fraction of people in town attending that church.

A. Now find the equilibrium choice of effort for churches A and B assuming that their objective function is $f - \frac{2}{3}e^2$.

Each church puts forth effort equal to 3/8. If they were the only church in town their effort would rise to ¾.

B. Suppose each church as objective function $f - \alpha e^2$, where alpha is an unknown number. Consider the case there is only one church in town, suppose that it is located at point zero. What value of α is consistent with this church deciding to put out enough effort that exactly one fourth of the people in town go to church? Would the answer be different if the church were at point 1?

For church A at zero the objective function is $e - \alpha e^2$. *The first order condition is* $1 = 2\alpha e$. *We are looking for the outcome where the fraction of the town going to A (which equals e) is ¼, so plugging in ¼ for e yields* $\alpha = 2$. *The same is true for a church at point 1.*

C. Suppose again there are two churches in town, at 0 and 1. Suppose their objective function was $f - \alpha e^2$, where α takes on the value that you solved for in part B. If α takes on this value, than will these churches "overlap" in equilibrium (in other words, will everyone in town go to church)?

No. Each church will set e = ¼ and half the people will go to church and half will not.