

Problem Set 6:

1. Suppose that there is a town of people living along a line from zero to 1. The closer you live to point 0, the more you like choir music at church, the closer you live to point 1 the less you like choir music. Your utility from going to church c is $U = e_c - |x - x_c|$. If you don't go to church your utility is zero. If you go to church, you'll go to the church that gives you the highest utility. People in the town are 'evenly spread out' so that half the people live between zero and $\frac{1}{2}$, and so on.

Suppose there were 2 churches in this town. To make life easy let's assume that both churches have their effort levels set to 1. Also, let's suppose that there are 101 possible locations x_c that a church could choose to be out evenly spread out through the town. So x_c could be 0, 0.01, 0.02, all the way up to 1. Finally, let's suppose that each church wants as many members as possible.

- A. Verify to yourself that if there was just one church in town with effort level 1, then everybody would go to church no matter where the church was located.
- B. Now let's think about 2 churches in town. Suppose the first church was located at point zero and church two was deciding where it would locate itself (that is how much choir music it would use). Where should church 2 go given that church 1 is at zero?
- C. Now suppose church one was at $\frac{1}{4}$ and church 2 had to decide where to locate. Where would it locate in this case?
- D. Now suppose that church 1 was located at 0.49. Where would church 2 locate now?
- E. In parts B, C, and D, would church 1 be happy about things once church 2 made it's decision? Or would church 1 want to change what it was doing in each case? What do you think the Nash Equilibrium here is?
- F. The above problem is a famous one created to explain why similar stores (e.g., gas stations, hardware stores, etc) locate near each other in a town. But in our case spatial distance is a metaphor for religious content. Suppose there was a town with 2 churches. Based on this problem would you expect these churches to have very similar content or very different religious content from each other?

2. Suppose there is a town of people located along a line from zero to 1. Suppose that there are two churches in the town located at 0 and 1. As usual let the utility from going to church c be

$U = e_c - |x - x_c|$. In class we assumed that each church maximized $f - \frac{1}{2}e^2$, where f is the fraction of people in town attending that church.

A. Now find the equilibrium choice of effort for churches A and B assuming that their objective function is $f - \frac{2}{3}e^2$.

B. Suppose each church has objective function $f - \alpha e^2$, where alpha is an unknown number. Consider the case there is only one church in town; suppose that it is located at point zero. What value of α is consistent with this church deciding to put out enough effort that exactly one fourth of the people in town go to church? Would the answer be different if the church were at point 1?

C. Suppose again there are two churches in town, at 0 and 1. Suppose their objective function was $f - \alpha e^2$, where α takes on the value that you solved for in part B. If α takes on this value, than will these churches “overlap” in equilibrium (in other words, will everyone in town go to church)?