Problem Set 5: Economics and Religion Models

1. Suppose that an individual's utility was $\ln c_1 + \ln c_2 + a$, where after life consumption *a* is a function of religious attendance in each period of life: $a = \ln r_1 + \ln r_2$. The person's budget constraint is as in class: $pc_1 + s_1 = w_1(1 - r_1)$ for period 1 and $pc_2 + w_2 r_2 = w_2 + (1+i)s_1$ for period 1.

   A. Combine the two budget constraints as in class

   $$pc_1 + w_1 r_1 + \frac{pc_2}{(1+i)} + \frac{w_2 r_2}{(1+i)} = w_1 + \frac{w_2}{(1+i)}$$

   B. Set up the Lagrangian

   $$L = \max_{c_1, c_2, r_1, r_2} \ln c_1 + \ln c_2 + \ln r_1 + \ln r_2 + \lambda\left(w_1 + \frac{w_2}{(1+i)} - pc_1 - w_1 r_1 - \frac{pc_2}{(1+i)} - \frac{w_2 r_2}{(1+i)} \right)$$

   C. Find solutions for consumption and religion as functions of wages, the interest rate, and the price of consumption.

   The first-order conditions are:

   $$\frac{1}{c_1} = p\lambda \qquad \frac{1}{r_1} = w_1\lambda$$

   $$\frac{1}{c_2} = \frac{p\lambda}{(1+i)} \qquad \frac{1}{r_2} = \frac{w_2\lambda}{(1+i)}$$

   Solve out for (say) c_1 *and plug into the budget constraint, and you'll get that*

   $c_1 = \frac{1}{4p}\left(w_1 + \frac{w_2}{1+i} \right)$. *The part in parentheses, you'll notice, is basically your "lifetime wealth" as of period 1. So you'll spend a fourth of your wealth on* c_1. *Solving out for the other variables is trivial using the first order conditions and the solution for* c_1.

   $$c_2 = \frac{1+i}{4p}\left(w_1 + \frac{w_2}{1+i} \right), \quad r_1 = \frac{1}{4w_1}\left(w_1 + \frac{w_2}{1+i} \right), \quad r_2 = \frac{1+i}{4w_2}\left(w_1 + \frac{w_2}{1+i} \right)$$

   D. The above problem is for 2 periods. Now suppose that this individual lives for 1,033 years. Utility is $a + \sum_{t=1}^{t=1,033} \ln c_t$, so it is just like the utility function before but now there are more periods. And after life consumption is similar to before too: $a = \sum_{t=1}^{t=1,033} \ln r_t$. As before, the individual can save money from one period to the next. Let's assume that $w_t = 1$ each period, that $p = 1$ each period, and that *i* is zero. Will the attendance and consumption choices be the same in each period? Solve for what consumption and attendance will be in any given period *t*.

This is an interesting question in that you almost have to use some economic intuition to solve it. Your intuition right away might be that, if i=0, then we get $c_1 = c_2$ *in the two-period problem from part C, so we should get it here too.*

That is correct. The trickiest part maybe is writing down the budget constraint. There are a few ways to figure it out but one way is to start in the last period. We have

$pc_{1033} = w_{1033}(1-r_{1033}) + s_{1032}(1+i)$, or

$c_{1033} = 1 - r_{1033} + s_{1032}$, or

$c_{1033} + r_{1033} - 1 = s_{1032}$.

For the period before it is

$c_{1032} + s_{1032} = (1-r_{1032}) + s_{1031}$.

Plug in for s_{1032} *into this budget constraint and you get*

$c_{1032} + c_{1033} + r_{1033} - 1 = (1-r_{1032}) + s_{1031}$,

or $c_{1032} + c_{1033} + r_{1033} + r_{1032} - 2 = s_{1031}$

Keep plugging in forever and you'll get

$c_1 + c_2 + \ldots + c_{1033} + r_1 + r_2 + \ldots + r_{1033} = 1{,}033$.

Which, if one thinks about it a bit, kind of had to be the right answer from the start.

The Lagrangian is $\sum_{t=1}^{t=1,033} \ln r_t + \sum_{t=1}^{t=1,033} \ln c_t + \lambda \left(1033 - \sum_{t=1}^{t=1,033} c_t - \sum_{t=1}^{t=1,033} r_t \right)$

Take the derivative for c_t *and* r_t *and you will see that* $1/c_t = 1/r_t = \lambda$ *for all t. So c and r are the same forever. Plug into the budget constraint and you find* $c_t = r_t = 1/2$.

2. Suppose that an individual got utility from beer *B* and donations to church *D*. Two people go to church, and *D* is the sum of their donations: $D = d_1 + d_2$. Person 1's utility function is:

$$U = 2B^{1/2} + 2(d_1 + d_2)^{1/2}$$

and their budget constraint is $pB + d_1 = I$

A. Set up person 1's Lagrangian

$$L = \max_{B,d_1} 2B^{1/2} + 2(d_1 + d_2)^{1/2} + \lambda(I - pB - d_1)$$

B. Taking person 2's donation as given, solve for d_1 and *B*. Your answers will be a function of *I*, *p*, and d_2 . If person 2 donates more to the church; does person 1 donate more? Do they drink more beer?

The solutions are $B = \dfrac{I + d_2}{p + p^2}$ *and* $d_1 = \dfrac{I - (d_2 / p)}{\frac{1}{p} + 1}$. *As person 2 donates more, person 1 donates less and spends the money on beer!*

C. Suppose that in the Nash Equilibrium each person donates the same amount, and that for each person prices and income are 1. Solve for each person's donations and beer.

$B^* = \frac{2}{3} \; and \; d_1^* = \frac{1}{3}$

D. Plug your answer from part C into the utility function (use a calculator) and see what each person's utility is. *The answer is ~ 3.266*

E. Now let's pretend we could order people in this church around and they had to obey us. Let's see if we can do better than what we got for parts C and D. Let's maximize the "added up" utility functions:

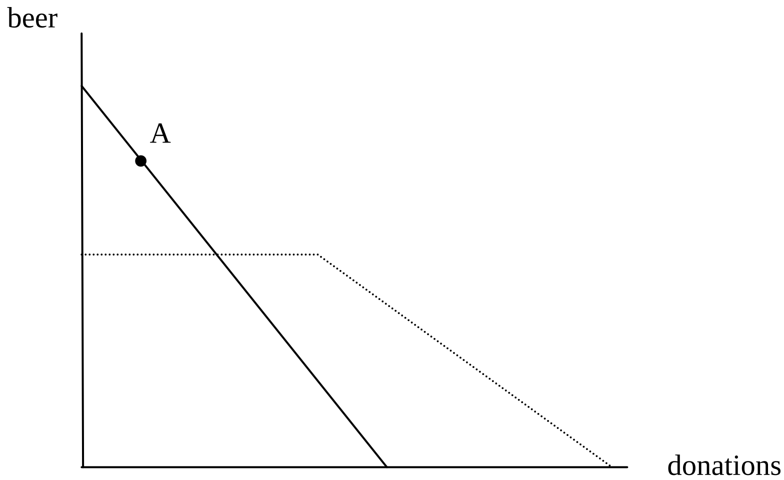
$$2B_1^{1/2} + 2(d_1 + d_2)^{1/2} + 2B_2^{1/2} + 2(d_1 + d_2)^{1/2}$$

The first two terms are person 1's utility and the next two terms are person 2's utility. The budget constraint is $p(B_1 + B_2) + d_1 + d_2 = I + I$. But to make life easy let's assume I =1 and p = 1 too. Solve for d_1, d_2, B_1, and B_2. Plug in and verify that people are better off here than they were in part D. What is going on here?

Now $B_1^* = B_2^* = \frac{1}{3}$ *and* $d_1^* = d_2^* = \frac{2}{3}$*, and each person's utility is ~3.465. What is going on is that when each person donates in part C, they are not thinking about how their donation helps others. But here in part E we *are* thinking about that, so we make each person donate more, and consequently everybody is better off. This illustrates the problem for congregations that we discussed in the Iannaccone model—that people may not thinking enough about the positive effects their religious participation has on others.*

3. Suppose somebody is thinking of joining a strict church. They consume beer and donate money to the church. The point A below shows the outcome they would choose with no church. The dashed budget line shows the budget line they would face if they joined the church.

Here is the question: what sort of indifference curve passing through point A would make a person more likely to join the church—would it be a "straight" sort of indifference curve or more of a "right angle?" Iannaccone's theory suggests that strict churches should attract people who either (a) view drinking and donations as activities that substitute well for each other or (b) activities that don't substitute well for each other. Which do you think he finds? Is this intuitive?



Obviously, a flatter indifference curve has a better chance of falling "inside" of the new budget line. This means that the more substitutable donations are for beer, the more likely it is that people will want to join a church offering the new budget line. This is what Iannaccone finds.

To me, anyways, this is not intuitive. The idea that strict churches would especially appeal to people who view strict church life as substitutable for secular consumption is a bit surprising.

But Iannaccone argues that it makes some sense. People with very flat indifference curves can be very sensitive to price changes. With perfect substitutes, for instance, indifference curves are straight lines and a small price change might make you move from one corner solution to the other. Iannaccone says that if people with flat indifference curves join strict churches, than we should see radical responses to price changes from these people—they will have very dramatic conversions, or they will suddenly leave. In other words, people don't leave a strict church (like a cult) and start going to a moderately strict church (like maybe the Catholic Church). Instead, they'll leave and be atheists. There is some evidence for that sort of radical-change story in the data, in fact.

This prediction illustrates how using an economic model can produce pretty surprising predictions. Iannaccone's model makes a bunch of predictions, it has been cited in hundreds of papers and I have never seen a serious challenge to it. Pretty cool!