Problem Set 3
Public Economics
Professor Hungerman

1. Basic Nash Equilibrium
This is a classic economic game called the prisoner's dilemma. Suppose two criminals, Bonnie and Clyde, rob a bank. Later they are caught by the cops and taken to headquarters for interrogation. They are put in separate soundproof rooms. A detective walks into each criminal's interrogation room and makes the following offer:

"At the moment the police only have enough evidence to arrest you for a minor crime that would put you in jail for 1 year. The penalty for bank robbery is 10 years in jail. If you testify against your partner so that they are convicted of bank robbery, I'll knock 1 year off any sentence you receive if you get convicted."

Bonnie and Clyde each know the only way they will get convicted of bank robbery is if their partner rats them out and agrees to testify. So their payoffs look like those in the below table, where each cell shows the pair (Clyde's outcome, Bonnie's outcome) depending on who testifies against their partner and who doesn't.

		Bonnie	
		Don't Testify	Testify
Clyde	Don't Testify	(1 year in jail, 1 year in jail)	(10 years in jail, 0 years in jail)
	Testify	(0 years in jail, 10 years in jail)	(9 years in jail, 9 years in jail)

Let's assume that both Bonnie and Clyde view more time in jail as a bad thing

A. Are any of the 4 outcomes above Pareto Efficient?

All of the outcomes are PE except for the outcome (*Testify*, *Testify*). *That outcome is dominated by* (*Don't Testify, Don't Testify*)

B. Find the outcomes in the Nash Equilibrium(s) of this game

The Nash Equilibrium is (*Testify, Testify*)*—that is, the equilibrium is the one outcome that is not PE.*

C. Can you think of any real-life situations that might be similar to the above scenario?

The Prisoner's Dilemma is not just interesting because it has an equilibrium which is not PE, but because the intuition seems appropriate for a number of circumstances. Just one example is nuclear arms races. Probably a Pareto Efficient outcome would be if no countries had nuclear weapons. But every country thinks to itself "gosh, I hope nobody else has nukes, but it would be great if I had them." So everyone makes them and we end up in a not PE outcome (so in this example "don't testify" is like "don't make nukes" and "testify" is like "make nukes").

D. (Have you ever seen the tv show Batman the animated series? Do you know the villain Twoface?). Suppose that bonnie is crazy and when given a decision she flips a coin. If the coin says heads, she testifies. If it says tails she refuses to testify. Suppose that Clyde knows bonnie will make her decision this way. Does that change what Clyde's decision should be?

This will not change Clyde's strategy. No matter what Bonnie does Clyde's best response is to testify.

2. Consider a community with *n* individuals. Each individual has the same utility function:

$$U\left(x,G\right)=\ln(x)+\ln(G)$$

where *x* is a private good and *G* is a public good. The consumer's budget constraint for consumer *i* is $x_i+g_i=m-\tau$, where taxes τ and endowed income m are assumed to be the same for everyone. The public good is provided by a constant-returns-to-scale technology so that $G=\sum_{i=1}^{n}g_i+\sum_{i=1}^{n}\tau$.

A. What is a public good?

Public goods are non rivaled in consumption and non excludable. You should know what those terms mean!

B. Set up the Lagrangian for individual *i* and find *i*'s best response function

The lagrangian is

$$\max_{x,g_i}=\ln(x_i)+\ln(g_i+\sum_{j\neq i}g_j+n\tau)+\lambda(m-\tau-g_i-x_i)$$

From the First Order Conditions one can find

$$x_i=g_i+\sum_{j\neq i}g_j+n\tau=G$$

Plug this into the budget constraint

$$x_i+g_i=g_i+\sum_{j\neq i}g_j+n\tau+g_i=m-\tau$$

The best response function is

$$g_i=(1/2)*[m-(n+1)\tau-\sum_{j\neq i}g_j]$$

C. Solve for the equilibrium level of the public good. Does it depend on taxes?

Now we use the shortcut discussed in class were we not that in equilibrium $g_i = g_j = g^*$ *then the above becomes*: $(n+1)g^* = m-(n+1)\tau$. *So* $g^* = m/(n+1)-\tau$

Thus $G^* = ng^* + n\tau = (nm)/(n+1)$ *, which does not depend on taxes.*

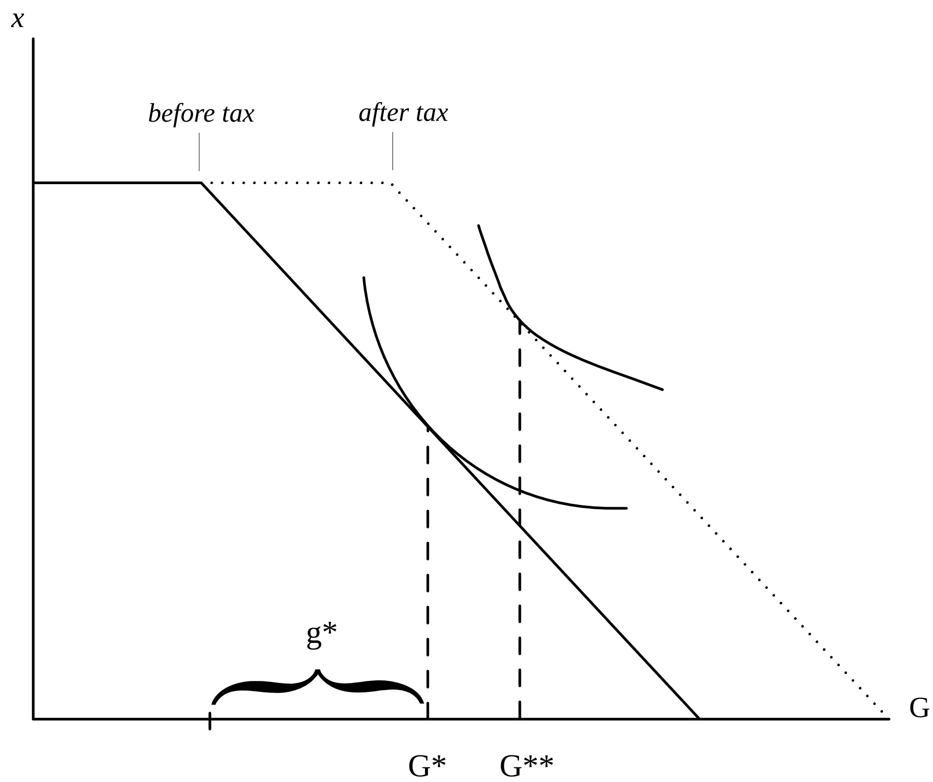
D. Draw a picture of the equilibrium, with budget constraints and indifference curves, as we did in class.

The picture—at first—should be very close to what was in class.

E. Using the picture you just drew, consider the following scenario. Suppose a new person, person *n* + 1, moved into town. This person is different from everyone else and in particular this person donates nothing to the public good and (at first) pays no taxes. Then one day the government taxes this person, and uses the revenue to contribute to the public good. How is this different from D? Assuming that the *n* individuals contributing to *G* view the public good as a strictly normal good, explain how taxing person *n* + 1 might affect the equilibrium level of *G*.

Before when we raised taxes on people they just reduced their voluntary provision of the public good. Now, however, as person n + 1 *is taxed person i can still consume the same amount of c as before and G will go up! In other words, the budget constraint shifts out.*

If G is normal, than person i will prefer a new equilibrium where crowd out is higher than before. See the picture on the next page, where G is the old level of the public good and G** is the new level.*



x
before tax
after tax
g*
G
G*
G**