

Problem Set 1
Public Economics
Professor Hungerman

1. Suppose that an individual consumes two goods, x and y , and the individual's utility function can be written as

$$U(x, y) = \alpha \ln x + (1 - \alpha) \ln y,$$

where $0 < \alpha < 1$. This individual maximizes utility subject to the constraint:

$$p_x x + p_y y = m$$

A. Verify that the marginal utility of x is positive, but falls as x increases. Does this make sense? What is the case for y ?

The marginal utility of x is $\frac{\partial U}{\partial x} = \frac{\alpha}{x}$, which is greater than zero since $0 < \alpha < 1$. As x gets bigger, however, the marginal utility falls towards zero. This does make sense; these preferences satisfy the law of diminishing marginal utility. The same is true for y .

B. Write down the individual's maximization problem—that is, write down the Lagrangian.

The maximization problem is

$$\max_{x,y} \alpha \ln x + (1 - \alpha) \ln y + \lambda(m - p_x x - p_y y)$$

C. Take the derivative of your answer in A with respect to x , y , and the Lagrange multiplier λ , to get three first order conditions.

The three first order conditions are

$$\text{for } x: \frac{\alpha}{x} - \lambda p_x = 0$$

$$\text{for } y: \frac{1 - \alpha}{y} - \lambda p_y = 0$$

$$\text{for } \lambda: m - p_x x - p_y y$$

D. Using these first order conditions find the *demand equations*: that is, solve for x and y as functions of α , p_x , p_y , and m .

Use the first two equations to show $p_x x = \frac{\alpha}{1 - \alpha} p_y y$. Plug this into the budget equation to find

$$x^* = \frac{\alpha m}{p_x}, y^* = \frac{(1 - \alpha)m}{p_y}.$$

E. What fraction of income will this individual spend on x ? Does this fraction depend upon p_x or m ?

This individual spends a fraction α of his income on x , regardless of what income is and what prices are. (To see this, use the answers from part D to calculate $p_x x^ / m$). These preferences are sometimes called “Constant Expenditure Share” preferences, because a constant share of your income is spent on each good.*

F. What is the price elasticity of demand for good x ?

Calculate the elasticity as
$$\frac{\partial x}{\partial p_x} \frac{p_x}{x} = \frac{-\alpha m}{p_x^2} \frac{p_x}{\left(\frac{\alpha m}{p_x}\right)} = -1.$$

Now here is a question for you—looking back to the answer to part E, could you have guessed this to be true without doing any math at all?

2. Many studies in the social sciences have shown that religious individuals look different from other individuals. For example, people who go to church are less likely to smoke, be heavy drinkers, and commit crimes than are other people. Some individuals have argued that this evidence indicates a positive benefit from attending church, and policy makers should consider this when making laws which could affect religion. How could the above studies be biased?

The problem with these studies is that individuals who are religious might be different from other individuals in many ways. For instance, religion may have no impact on your well being. Instead their might be two types of people in the world “good” people and “bad” people, and “good” people want be good by going to church, and also by being law abiding nonsmokers. So church does nothing here, it is just a symptom of goodness.

The ideal solution to this problem would be a randomized trial where a large group of individuals is randomly assigned to be religious and a control group is randomly assigned to not be religious, and we would see how the behaviors of the two groups differ. Of course such an experiment is impossible. The effects of religion remain a big unanswered question in the social sciences, although some progress on the topic has been made in recent years.

3. What is the poverty line for a family of 5 in the United States right now?

Try googling the words “poverty line.” The Department of Health and Human Services, at <http://aspe.hhs.gov/poverty/09poverty.shtml>, gives this as the poverty line for 5: \$25,790.

4. In class we discussed whether as a family grows bigger the marginal cost of having a child increases or decreases. Suppose the poverty line for a family of size x is made by taking the poverty line of a family size $x-1$ and adding in the marginal cost of one more family member. If that was the case, do the real poverty guidelines indicate that the marginal cost of one more child is rising or falling for poor families?

The Poverty Guidelines add \$3,740 for each person—so they assume the marginal cost of each child is constant. There is actually some debate about whether the government should be more careful when making the poverty guidelines regarding this issue.

5. Turn in the abstracts of the three papers you are interested in learning about, as discussed in class and on the syllabus.

Go do this!