and

$$\text{Tr}(W_k M_s) = \text{Tr}[(\sum_{m,n} w_n |c_m^n|^2 |\psi_m^k\rangle\langle\psi_m^k|)(\sum_j a_j^s |\psi_j^k\rangle\langle\psi_j^k|)]$$

$$= \text{Tr}(\sum_{m,n,j} a_j^s w_n |c_m^n|^2 |\psi_m^k\rangle\langle\psi_m^k|\psi_j^k\rangle\langle\psi_j^k|)$$

$$= \text{Tr}(\sum_{m,n} a_m^s w_n |c_m^n|^2 |\psi_m^k\rangle\langle\psi_m^k|)$$

$$= \sum_{m,n,t} a_m^s w_n |c_m^n|^2 \langle\psi_t^k|\psi_m^k\rangle\langle\psi_m^k|\psi_t^k\rangle$$

$$= \sum_{n,t} a_t^s w_n |c_t^n|^2 \ .$$

Thus, $\text{Tr}(W_{qm} M_s) = \text{Tr}(W_k M_s)$, and the theorem is proved.