# Basic Combinatorics

## Math 40210, Section 01 — Fall 2012

## Homework 4 — due Monday September 24

**General information**: Homework is an essential part of your learning in this course, so please take it very seriously. It is extremely important that you keep up with the homework, as if you do not, you may quickly fall behind in class and find yourself at a great disadvantage during exams.

You should treat the homework as a learning opportunity, rather than something you need to get out of the way. Reread, revise, and polish your solutions until they are correct, concise, efficient, and elegant. This will really deepen your understanding of the material. I encourage you to talk with your colleagues about homework problems, but your final write-up must be your own work.

You should present your final homework solutions clearly and neatly. Keep in mind that when you write a homework solution, you are trying to communicate the solution to someone other than yourself, so incomplete sentences and personal shorthand is not helpful!

Due to manpower issues, I will only grade selected homework problems, but I plan to quickly post solutions to all the problems soon after I've collected them up.

**Reading**:

- Section 1.4.1
- Section 1.4.2
- Section 1.4.3, up to the statement of Theorem 1.23
- Slides on **P** versus **NP** from the course website
- Introduction to Section 1.5
- Section 1.5.1
- Section 1.5.2

**Problems**:

- Section 1.4.2: 2, 4, 5, 7 (for 7, remember that $K_{n_1,n_2}$ is defined for all $n_1, n_2 \geq 1$)
- Is the converse to 1.4.2(5) true? That is, is it true that if $G$ is Eulerian, then every edge of $G$ lies on an odd number of cycles?
- Section 1.4.3: 1, 3, 10, 12 (for question 10b, a graph being *traceable* means that it has a Hamilton path)
- Show that the Petersen graph (Figure 1.63) has a Hamilton path but not a Hamilton cycle
- **P** versus **NP**: Both of these are necessarily informal questions:
  - Explain how you could fairly quickly check whether a given graph is connected. Assume for concreteness that you are given the graph in the form of its adjacency matrix, and that you can perform matrix multiplication quickly. (This question is asking you to justify why the property of being connected is in **P**)
  - A *bridge* in a graph is an edge whose deletion increases the number of components by 1. I give you a graph which I guarantee is connected. Explain how you could fairly quickly check whether or not it has a bridge
- Section 1.5.1: 1, 4, 5, 8 (for 4, you have to be careful. It's not enough to say, for example, "remove an edge attached to a leaf; draw the smaller tree in the plane without crossing edges (ok by induction), then put in the last edge". If you take this approach, you need to give a convincing argument that the edges of the smaller tree have been drawn in such a way that a new edge can be added without crossing one of the previous edges. Also, for this question you can't quote Euler's formula)