Math 30530, Probability

Quiz 4, Wednesday April 3

Solutions

1. This question concerns a random variable X that is Exponential with parameter 4.

   (a) Write down the density function of X.
   Solution:
   $$f(x) = \begin{cases} 4e^{-4x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0. \end{cases}$$

   (b) Find that number $x^\star$ such that $P(X \leq x^\star) = \frac{1}{2}$ (so also, $P(X \geq x^\star) = \frac{1}{2}$ — this number $x^\star$ is called the *median* of X).
   Solution: $P(X \leq x^\star) = 1/2$ means $\int_0^{x^\star} 4e^{-4x}\, dx = 1/2$, or $[-e^{-4x}]_{x=0}^{x^\star} = 1/2$, or $1 - e^{-4x^\star} = 1/2$, or $e^{-4x^\star} = 1/2$, or $e^{4x^\star} = 2$, or $4x^\star = \log 2$, or
   $$x^\star = \frac{\log 2}{4} \approx 0.173.$$

   (c) Are $x^\star$ and $E(X)$ the same?

   Solution: $E(X) = 1/4$ so $x^\star$ and $E(X)$ are *not* the same.

2. A random variable Y that models choosing a random number between 0 and 2, that favours numbers that are closer to 1, has density
   $$f(x) = \begin{cases} 0 & \text{if } x < 0 \text{ or } x > 2 \\ cx(2-x) & \text{if } 0 \leq x \leq 2. \end{cases}$$

   (a) What is c?
   Solution: We require $\int_0^2 cx(2-x)\, dx = 1$ for f to be a valid density, or $4c/3 = 1$, or $c = 3/4$.

   (b) What is the expected value of X?
   Solution:
   $$E(X) = \int_0^2 \frac{3}{4} x(2-x)\, dx = 1.$$