# Math 30530, Probability

Quiz 1, Wednesday February 13

Solutions

1. $A$ and $B$ are events in a sample space $S$, with $P(B) > 0$. Give the definition of *the conditional probability of $A$, given $B$.*

   **Solution**: The conditional probability of $A$ given $B$, $P(A|B)$, is defined by

   $$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

2. I toss a fair coin 3 times.

   (a) What is the probability of getting at least 2 heads?
   **Solution**: The sample space is $S = \{hhh, hht, hth, thh, htt, tht, tth, ttt\}$, all 8 outcomes equally likely. The event {at least two heads} has four sample points in it: $hhh, hht, hth$, and $thh$. So

   $$P(\text{at least two heads}) = \frac{4}{8} = 0.5.$$

   (b) Given the information that *at least one* of the tosses was heads, what is the conditional probability of getting at least 2 heads?
   **Solution**: Given the information that at least one of the tosses was heads, the (conditioned) sample space reduces in size to 7 ($ttt$ is removed). The event {at least two heads} has all four of its sample points in this reduced sample space. So

   $$P(\text{at least two heads}|\text{at least one head}) = \frac{4}{7} = 0.5714\ldots.$$

   (c) Given the information that there were at least two heads, what is the conditional probability that the first toss is tails?
   **Solution**: Given the information that at there were at least two heads, the (conditioned) sample space reduces in size to 4 ($\{hhh, hht, hth, thh\}$). The event {first toss tails} one of its sample points in this reduced sample space ($\{thh\}$). So

   $$P(\text{first toss tails}|\text{at least two heads}) = \frac{1}{4} = 0.25.$$

   (d) Are the events {at least two heads} and {first toss heads} independent?
   **Solution**: We have computed $P(\text{first toss tails}|\text{at least two heads}) = 0.25$, so by

   $$P(A^c|E) = 1 - P(A|E)$$

   we have $P(\text{first toss heads}|\text{at least two heads}) = 0.75$. But

   $$P(\text{first toss heads}) = \frac{|\{hhh, hht, hth, htt\}|}{8} = 0.5.$$

   Since $P(\text{first toss heads}|\text{at least two heads}) \neq P(\text{first toss heads})$, these two events are *not* independent.