Introduction to Probability, Fall 2013

Math 30530 Section 01

Homework 2 — due in class Friday, September 13

General information

At the top of the first page, write your name, the course number and the assignment number. If you use more than one page, you should **staple all your pages together**. The grader reserves the right to leave ungraded any assignment that is disorganized, untidy or incoherent.

Reading

- Sections 1.3, 1.4

Problems

1. Chapter 1, problem 14 ("doubles" means that both dice show the same number).

2. Chapter 1, problem 16.

3. Chapter 1, problem 19.

4. The top shelf of the cereal aisle in the grocery store has 6 boxes of nice cereal and 3 boxes of nasty cereal. The middle shelf has 4 nice boxes and 4 nasty boxes. The bottom shelf has 2 nice boxes and 3 nasty boxes. I choose one box, completely at random, from each shelf (three boxes in all).

   (a) What's the probability that among the three boxes I choose, one is a nice box from the top shelf?

   (b) Given the extra information that I choose exactly two nice boxes in all, what now is the probability that among the three boxes I choose, one is a nice box from the top shelf?

5. I pick three cards at random from a deck of 52 cards, one after the other.

   (a) Compute the probability that the third card is a spade, given that the first two were spades.

   (b) Compute the probability that the *first* card is a spade, given that the *last* two were spades.

6. I read the following in a newspaper: "An ectopic pregnancy is twice as likely to develop when the pregnant woman is a smoker, as when she is a non-smoker". I read in a survey that "32% of all women of childbearing age are smokers". From these two facts, figure out what percentage of women having ectopic pregnancies are smokers.

7. Andy Murray and Rafael (Rafa) Nadal play a best-of-five-set tennis match (that is, they keep playing sets until one of them has won three; that person is then the winner). Each player has a .5 chance of winning each set, regardless of what happened on previous sets (so, for example, the probability that Andy wins the third set, given that Rafael won the first and Andy the second, is .5). I watch the match, and observe who won each set.

   (a) Write down Ω, the sample space for this experiment. Use the following notation: $AARA$ stands for the outcome that Andy wins the first set and the second, Rafa the third, and Andy the fourth. **Hint**: your sample space should include exactly 20 outcomes.

   (b) **Use the multiplication rule** to figure out the probability of each of the outcomes: RRR, $AARA$, and $RAARR$. Use common sense to then write down the probabilities of each of the other outcomes. **Hint**: the sum of your 20 answers should be 1.

   (c) Calculate the probability that "Rafa wins the match (gets to three sets before Andy does)"

   (d) Calculate the probability that "Rafa has a two sets to one advantage after three sets"

   (e) Calculate the *conditional* probability that "Rafa wins the match, given that he has a two sets to one advantage after three sets"

8. When I go away for a week, I ask my neighbor to regularly water my plants. I know from experience that she is 90% likely to actually do this. If my plants are regularly watered, they stay healthy with probability .9. If my plants are not regularly watered, they stay healthy with probability .2.

   (a) What is the probability that my plants will be healthy when I return from my trip?

   (b) I come home from my trip, and find that my plants are not healthy. What is the probability that my neighbor *failed* to come by to regularly water them?

9. Professor Bunsen always starts his Alchemy 231 lecture course with one of the three great alchemical experiments: turning lead into gold (20% of all times that he teaches the course), brewing the elixir of life (40% of the times) and creating the Philosopher's stone (40% of the time). When he tries to turn lead into gold, the result always ends with a explosion; when he brews the elixir of life, there is a 50% chance of an explosion, and when he creates the Philosopher's stone, 8 times out of 10 there is an explosion. Dean Crawford wants to see which experiment Professor Bunsen will do this year, but he arrives late. If he see the lecture-hall filled with post-explosion smoke, what should

he conclude is the probability that he has just missed a demonstration of brewing the elixir of life?

10. 98% of the time in which a person is telling the truth, a lie detector correctly detects this. 95% of the time in which a person is lying, the lie detector correctly detects this. An operator knows from experience that when suspects are being interrogated, they lie two thirds of the time. Based on this information, what would you say is the probability that a statement detected as a lie is in fact a lie? (Let D be the event that a statement is detected as a lie, T the event that a statement is not a lie and L the event that a statement is a lie).

11. Experience suggests that when a company sends a survey out, the response rate among under-25's is 30% and the response rate among over-45's is 55%. A survey is sent out to a bunch of people; 30% of the people are under-25's, 30% are over-45's, and the rest are 26-to-44's. There is a 40% response rate. Assuming that the response rates for this particular survey are typical, what do you think is the usual response rate from 26-to-44's?

12. If $\Pr(A) > 0$, show that $\Pr(A \cap B|A) \geq \Pr(A \cap B|A \cup B)$.

13. For events A and B with $\Pr(A) > 0$, $\Pr(B) > 0$, we say that A is *positively correlated* with B if $\Pr(A|B) > \Pr(A)$ (in other words, B occurring *increases* the chance that A occurs). If $\Pr(A|B) < \Pr(A)$ then A is *negatively correlated* with B. (If $\Pr(A|B) = \Pr(A)$ then A is *not correlated* with B.)

   (a) Show that if A is positively correlated with B, then B is positively correlated with A; in other words, show that if $\Pr(A|B) > \Pr(A)$ then $\Pr(B|A) > \Pr(B)$. (The same holds for negatively correlated and uncorrelated; so we can from now on just say "A and B are positively correlated", etc.)

   (b) Looking back at the earlier question about Andy Murray and Rafael Nadal, use your answers there to figure out whether the events "Rafa wins the match" and "Rafa has a two sets to one advantage after three sets" are positively correlated, negatively correlated, or uncorrelated.

   (c) I roll two dice and note the numbers that come up. Are the events "The sum of the numbers coming up is even" and "The two numbers are within one of each other" positively correlated, negatively correlated, or uncorrelated?

14. Chapter 1, problem 22. (Let A_i be the event that the ball drawn from the ith jar is white. Calculate $\Pr(A_1)$, and then use your knowledge of $\Pr(A_1)$ to calculate that $\Pr(A_2) = \Pr(A_1)$. Then show that if you know $\Pr(A_i) = \Pr(A_1)$, you also have that $\Pr(A_{i+1}) = \Pr(A_1)$. This is enough (by induction) to show that $\Pr(A_k) = \Pr(A_1)$ for every possible k.)