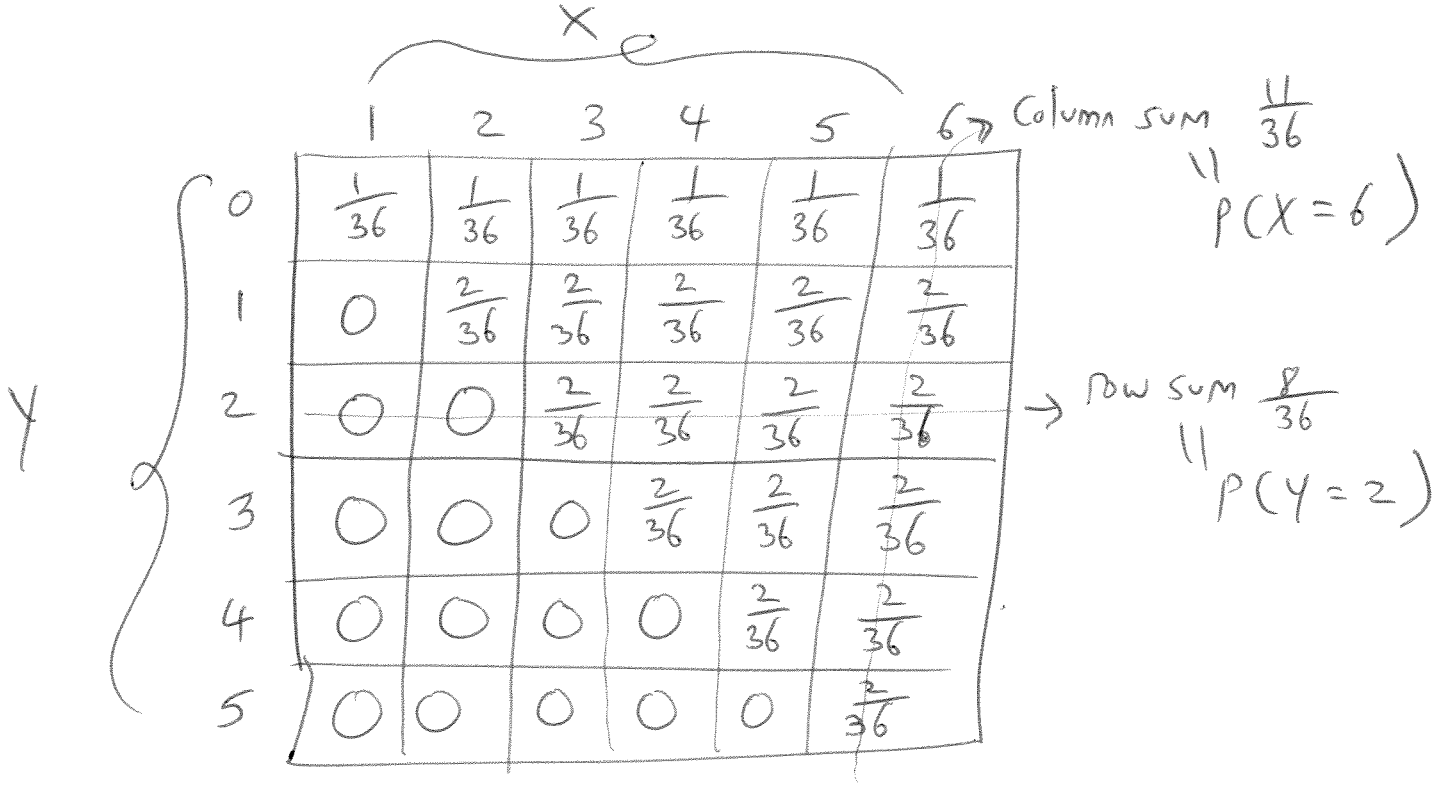
Joint mass of two random variables

1) Jack and Jill example

X = maximum in roll of 2 dice

Y = difference between the two rolls



- Entry corresponding to $X=x, Y=y$ is $P(X=x \text{ and } Y=y)$
- All entries add to 1

Joint mass: $p_{X,Y}(x,y) = P(X=x, Y=y)$

Joint CDF: $F_{X,Y}(x,y) = P(X \le x, Y \le y)$

Marginal mass of X: $p_X(x) = \sum_{\text{all possible } y} p_{X,Y}(x,y)$

Marginal CDF of Y: $F_Y(y) = \lim_{x \to \infty} F_{X,Y}(x,y)$

2) Alice and Bob example

Toss three coins

X = Bernoulli trial where success is the first two coins showing different faces

Y = same, with last two coins

Y \ X	1	0
1	$\frac{1}{4}$ (HTH, THT)	$\frac{1}{4}$ (HHT, TTH)
0	$\frac{1}{4}$ (HTT, THH)	$\frac{1}{4}$ (HHH, TTT)

In Alice and Bob example, X and Y are <u>independent</u>:

$$P(X=x, Y=y) = P(X=x)P(Y=y) \quad \text{for all } x, y$$

In Jack and Jill example, they are <u>not</u>:

eg, $P(X=3, Y=4) = 0$

$$\left.\begin{aligned} P(X=3) &= \frac{5}{36} \\ P(Y=4) &= \frac{4}{36} \end{aligned}\right\} \quad 0 \neq \frac{5}{36} \times \frac{4}{36}$$