Math 30530 — Introduction to Probability

Quiz 2 – Wednesday September 19, 2012

Solutions

1. There are three presses in my kitchen. The first two both contain ten boxes of quick-quinoa and 5 boxes of quick-couscous. Each of these boxes take ten minutes to prepare. The last press only has regular couscous, which takes 20 minutes to prepare. I reach into a randomly chosen press, select a random box, and prepare the contents for my dinner. Let X be the Bernoulli random variable where success is preparing couscous (regular or quick-) for dinner, and let Y be the number of minutes I spend preparing.

   (a) Write down the joint mass function of X and Y.

   Solution (5 pts): X has possible values 1 an 0, and Y has possible values 10 and 20. The joint mass function is the specification of all possible pair-probabilities $\Pr(X = x, Y = y)$. We have

   $$\Pr(X = 1, Y = 10) = \left(\frac{2}{3}\right)\left(\frac{1}{3}\right) = \frac{2}{9},$$

   because to have $X = 1$ and $Y = 10$ we must have picked quick-couscous: $2/3$ probability of picking one of the two presses with quick-couscous, and after that $1/3$ probability of actually sececting the couscous from the chosen press. We have

   $$\Pr(X = 0, Y = 10) = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9},$$

   by similar reasoning. We have

   $$\Pr(X = 1, Y = 20) = \frac{1}{3},$$

   because the only way to get 20 minute couscous is to choose press 3 (and take anything from it). Finally, we have

   $$\Pr(X = 0, Y = 20) = 0,$$

   because the only way to get 20 minute quinoa. For completeness we should say $\Pr(X = x, Y = y) = 0$ for all other pairs (x, y).

   (b) Are X and Y independent? Explain.

   Solution (2 pts): They seem to be dependent; if I give you the information $X = 0$, that tells you that Y must be 10, whereas if I give you no information about X, Y can be either 10 or 20; so information about X gives information about Y. To use the definition of independence, we could say that $\Pr(X = 0, Y = 20) = 0$ while $\Pr(X = 0)\Pr(Y = 20) = (4/9)(1/3) = 4/27$, so $\Pr(X = 0, Y = 20) \neq \Pr(X = 0)\Pr(Y = 20)$ and X and Y are not independent.

2. I'm rolling a dice five times, hoping to get three or more sixes. Exactly on of my first two rolls is a six. Given this information, what is the probability that I get three or more sixes in total?

   Solution (3 pts): Given that I have rolled one six in two tries, I need to roll either 2 or 3 sixes in the remaining 3 tries. The number of sixes I roll in the last 3 tries, X, is a binomial random variable with $n = 3$, $p = 1/6$, so

$$\Pr(X \geq 2) = \binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1 + \binom{3}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0 = \frac{2}{27}.$$