Math 30530 — Introduction to Probability

Quiz 1 – Solutions

1. 55% of students read the Observer daily, 25% live off-campus, and 63% either live off campus or read the Observer daily (or both). I pick a student at random (all students equally likely). What is the probability that the student I pick BOTH lives *on* campus AND reads the Observer daily?

   Solution: Let $R =$ reading observer, and $OC =$ living off-campus. Since

   $$\Pr(OC \cup R) = \Pr(OC) + \Pr(R) - \Pr(OC \cap R)$$

   we have $.63 = .25 + .55 - \Pr(OC \cap R)$, or $\Pr(OC \cap R) = .17$.

   We are looking for $\Pr(OC^c \cap R)$, which is $\Pr(R) - \Pr(OC \cap R) = .55 - .17 = .38$.

   Note: It is *not* correct to say $\Pr(OC^c \cap R) = \Pr(OC^c)\Pr(R)$; we don't know that the events of living off-campus and reading the observer are independent.

2. Use the definition of conditional probability and the three basic rules of probability to show that for any two events A and B (with $\Pr(B) > 0$),

   $$\Pr(A^c|B) = 1 - \Pr(A|B).$$

   Solution: $\Pr(A^c|B) = \Pr(A^c \cap B)/\Pr(B)$ and $\Pr(A|B) = \Pr(A \cap B)/\Pr(B)$ by definition, so we want to show

   $$\frac{\Pr(A^c \cap B)}{\Pr(B)} = 1 - \frac{\Pr(A \cap B)}{\Pr(B)}. \tag{1}$$

   Multiplying through by $\Pr(B)$, this becomes $\Pr(A^c \cap B) = \Pr(B) - \Pr(A \cap B)$ or $\Pr(B) = \Pr(A^c \cap B) + \Pr(A \cap B)$. This is true because B is the disjoint union of $A^c \cap B$ and $A \cap B$, and one of the basic rules of probability is that the probabilities of disjoint events add.

   Note 1: After reducing to (1), it is *not* correct to say $\Pr(A^c \cap B) = \Pr(A^c)\Pr(B)$ and $\Pr(A \cap B) = \Pr(A)\Pr(B)$, and cancel the $\Pr(B)$'s to reduce to $\Pr(A^c) + \Pr(A) = 1$: we don't know that the events A and B are independent.

   Note 2: Some people reasoned out the truth intuitively; I docked points for this because the question specifically asked for a proof using the definitions and basic rules.

3. I toss a dime and a nickel. Let A be the event that both coins come up showing the same side (either both heads or both tails). Let B be the event that the dime comes up tails.

   (a) Are A and B mutually exclusive?

   Solution: Listing the dime first, $A = \{HH, TT\}$ and $B = \{TH, TT\}$, so $A \cap B = \{TT\} \neq \emptyset$, so A and B are not mutually exclusive.

(b) Are A and B independent?

Solution: $\Pr(A) = 2/4 = 1/2$, $\Pr(B) = 2/4 = 1/2$ and $\Pr(A \cap B) = 1/4$. So $\Pr(A \cap B) = \Pr(A)\Pr(B)$, and the events *are* independent.