Introduction to Probability

Math 30530, Section 01 — Fall 2012

Homework 9 — due Monday December 3

General information: Homework is an essential part of your learning in this course, so please take it very seriously. It is extremely important that you keep up with the homework, as if you do not, you may quickly fall behind in class and find yourself at a disadvantage during exams.

You should treat the homework as a learning opportunity, rather than something you need to get out of the way. Reread and revise your solutions until they are correct and concise. This will help deepen your understanding of the material. I encourage you to talk with your colleagues about homework problems, but your final write-up must be your own work.

You should present your final homework solutions clearly and neatly. Keep in mind that when you write a homework solution, you are trying to communicate the solution to someone other than yourself, so incomplete sentences and personal shorthand is not helpful!

I plan to quickly post solutions to all the problems after I've collected them up.

Reading:

- Chapter 33
- Chapter 36
- Chapter 37
- Chapter 38

Problems: (**GW** indicates that the problem is taken from the course textbook by Gundlach and Ward)

1. During a season, a basketball team plays 70 home games and 60 away games. The coach estimates at the beginning of the season that the team will win each home game with probability .7, and each away game with probability .4, all games independent. *Estimate* the probability that, under these assumptions, the team wins at least 80 games in total during the season. (Don't try to calculate the exact probability; that's quite a pain).

2. Melons at the farmers market have a weight that is normally distributed with mean 3lbs and standard deviation .4lbs. The plastic bags that the melon vendor uses have breaking weight that is normally distributed with mean 10lbs an standard deviation .6lbs. Calculate the probability that is three melons are put into a bag, the bag will break (i.e., the breaking weight will be exceeded).

3. I keep flipping a coin until I have seen heads 200 times. *Estimate* the probability that this process will take me between 380 and 420 flips, inclusive (for this problem, you will want to use the continuity correction).

4. (a) Verify that if X and Y are independent exponential random variables, each with parameter λ, then $Z = X + Y$ has density function

$$f_Z(z) = \begin{cases} \lambda^2 z e^{-\lambda z} & \text{if } z \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

   (b) Suppose that X is an exponential random variable with parameter λ, and Y is a Gamma random variable with parameters r and λ, and that X and Y are independent. Verify that $Z = X + Y$ has density function

$$f_Z(z) = \begin{cases} \frac{\lambda^{r+1}}{r!} z^r e^{-\lambda z} & \text{if } z \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

   (This proves, by induction, that the formula for the Gamma density is correct.)

5. **GW** 33.2 (not part g))

6. **GW** 33.11

7. **GW** 33.15

8. **GW** 36.1

9. **GW** 36.10

10. **GW** 36.13

11. **GW** 37.3

12. **GW** 37.10 (the length of the shower is a random time)
13. **GW** 37.34
14. **GW** 37.38 (for both parts, what is sought is an approximate answer)
15. **GW** 37.49 (this question requires using a continuity correction)