Introduction to Probability

Math 30530, Section 01 — Fall 2012

Homework 7 — due Friday November 2

General information: Homework is an essential part of your learning in this course, so please take it very seriously. It is extremely important that you keep up with the homework, as if you do not, you may quickly fall behind in class and find yourself at a disadvantage during exams.

You should treat the homework as a learning opportunity, rather than something you need to get out of the way. Reread and revise your solutions until they are correct and concise. This will help deepen your understanding of the material. I encourage you to talk with your colleagues about homework problems, but your final write-up must be your own work.

You should present your final homework solutions clearly and neatly. Keep in mind that when you write a homework solution, you are trying to communicate the solution to someone other than yourself, so incomplete sentences and personal shorthand is not helpful!

I plan to quickly post solutions to all the problems after I've collected them up.

Reading:

- Chapter 25
- Chapter 26
- Chapter 28

Problems: (**GW** indicates that the problem is taken from the course textbook by Gundlach and Ward)

1. **GW** 25.1
2. **GW** 25.2
3. **GW** 25.8
4. **GW** 25.20 (both $0 \leq X \leq 1/2$ and $Y \leq 2X$ should happen simultaneously)
5. **GW** 26.4 (explicitly find the marginal densities here)
6. **GW** 26.7 (you might want to answer b and c before a here)
7. **GW** 26.8
8. **GW** 26.12
9. **GW** 28.1
10. **GW** 28.13
11. **GW** 28.21
12. (a) Do **28.22** when there is just **one** child.
    (b) Do **28.22** when there are just **two** children.
13. The following density is a special case of one that occurs fairly commonly in economics and social science. It's called the *Pareto* or *Zipf* density (Wikipedia has good pages on both). There's a Pareto density for each $\alpha > 1$, and it's given by

$$f_\alpha(x) = \begin{cases} 0 & \text{if } x < 1 \\ \frac{c}{x^\alpha} & \text{if } x \geq 1. \end{cases}$$

    Here c is a constant that depends on α.

    (a) For each $\alpha > 1$, find the value of $c = c(\alpha)$ that makes f_α a valid density.
    (b) For which values of α does the density f_α have a finite expectation?