Math 30530: Introduction to Probability, Fall 2012

Midterm Exam I

General information

When is the exam?

In class on Wednesday, October 10

What does the exam cover?

Ward and Gundlach, Chapters 1 through 22, but not Chapter 17

What is the format?

There will be six question; one of them will consist of short parts (true or false, fill in the blanks, ...) and the remaining questions will be free response

What can I do to prepare?

- Review the material! Know:
  - the basic objects of probability: experiments, outcomes, events, probability
  - how to manipulate events using union, intersection, complementation, De Morgan's laws
  - the basic rules of probability
  - the various consequences of the rules, including inclusion-exclusion
  - independence, conditional probability, Bayes' theorem
  - what a (discrete) random variable is, what its mass function and cumulative distribution functions are
  - what the joint mass function of two random variables is, and what it means for two random variables to be independent
  - the expected value of random variables, sums of random variables, and functions of random variables
  - the variance of a random variable

  - for each of the following named families of discrete random variables, know when to use a member of the family, what the parameters of the random variable are, what the possible values are, what the mass function is, and what the expectation and variance are:
    * Bernoulli
    * Binomial
    * Geometric
    * Negative Binomial
    * Hypergeometric
    * Discrete uniform
  - the basic rules of counting
  - the four fundamental counting problems: selecting r items from a set of size n, with and without replacement and with or without order mattering.

  As well as reviewing your class notes and the textbook (pay particular attention to the textbook's review chapters, 12 and 20), there are notes on counting and the named discrete random variables on the course page.

- Do plenty of questions! Basically any exercise from Chapters 1 through 21 (except Chapter 17) is appropriate; in particular there are review problems at the end of Chapters 12 and 20 (what's nice is that these chapters are reviewing multiple topics, so the title of the chapter doesn't immediately give away what the questions are testing; this replicates the exam situation). By Friday, all homework solutions should be up on the course website for you to review. At the end of this document I've included some more problems (from old exams) for you to look at.

- Come talk to me! I've tentatively set office hours for the following times:
  - Thursday 3.30-5.30, HH248 (this is a joint office hour with my 40210 class, who also have homework and an exam coming up. I request that if at all possible you come in the **first** hour; I'll be asking my 40210 students to come in the second).
  - Monday 4.15-5.30, HH248.
  - Tuesday 5-6, HH231.
  - Wednesday from 12.45 to class time, room 109 of the ROTC building.

Some review problems

1. 30 snowblowers, of which 7 have defects, are sold to a hardware store. The store manager inspects a total of 6 of the snowblowers randomly.
   (a) What is the probability that he finds no defective snowblowers?
   (b) What is the probability that he finds at least one defective snowblower?
   (c) What is the probability that he finds exactly two defective snowblowers?

2. Professor Bunsen always starts his Alchemy 231 lecture course with one of the three great alchemical experiments: turning lead into gold (20% of all times that he teaches the course), brewing the elixir of life (40% of the times) and creating the Philosopher's stone (40% of the time). When he tries to turn lead into gold, the result always ends with a explosion; when he brews the elixir of life, there is a 50% chance of an explosion, and when he creates the Philosopher's stone, 8 times out of 10 there is an explosion. Dean Crawford wants to see which experiment Professor Bunsen will do this year, but he arrives late. If he see the lecture-hall filled with post-explosion smoke, what should he conclude is the probability that he has just missed a demonstration of brewing the elixir of life?

3. A certain component in a (shoddy) computer typically fails 30% of the time, causing the computer to break. To counteract this appalling problem, a hacker decides to install n copies of the component in parallel, in such a way that the computer only breaks if all n components fail at the same time. Assuming that component failures are independent of each other,

   (a) find the probability that the computer does not break if $n = 3$ and

   (b) find the smallest value of n that should be chosen to ensure that the probability that the computer does not break is at least 98%.

4. A committee of 3 people is chosen from a group of 4 women and 3 women, with all such committees equally likely to be chosen. Let X be the number of women on the chosen committee.

   (a) Compute the mass function of X.

   (b) What is the expectation and variance of X?

   (c) What is the probability that X is at most 2?

5. (a) What is the probability that at least two of the six members of a family were born on a Saturday or Sunday? (Assume that all days of the week are equally likely as birth days.)

   (b) From the set of all families with four children, a child is selected at random and found to be a boy. Let X be the number of brothers the boy has. Compute the mass function, expectation and variance of X.

6. If four different fair dice are tossed, what is the probability that they will show four different numbers? (Here *fair dice* means a dice that shows each of its six faces with equal probability.)

7. Ten points are placed on the circumference of a circle. Two of them are selected at random. What is the probability that the two points are adjacent?

8. In how many arrangements of the letters "WEWILLWHALEONAIRFORCE" are the three W's adjacent?

9. Experience suggests that when sent a survey out, the response rate among under-25's 30% and the response rate among over-45's is 55%. A survey is sent out to equal numbers of under-25's, over-45's and 26-to-44's, and there is a 40% response rate. Assuming that the response rates for this particular survey are typical, what do you think is the usual response rate from 26-to-44's?

10. A woman gives birth to a child in the hospital's maternity ward. After a while, the child is brought into the hospital's nursery. Before the child is brought in, there are 5 boys and 10 girls in the nursery. Sometime after the child is brought in, a doctor walks into the nursery, picks a child at random, and notices that it's a boy. What's the probability that the woman gave birth to a boy? (You should assume that when a child is born, the probability that it is a boy is .5 and the probability that it is a girl is .5.)

11. 2% of all people who are qualified to apply for a position as administrative assistant at a mathematics department are familiar with the typesetting language LaTeX. How many qualified applicants should a department interview if it wants to be 50% sure that at least one of the applicants is familiar with LaTeX?

12. For any two events A and B, say whether each of the statements below are always true or sometimes false. If true, give a proof; if sometimes false, give an example based on the experiment of rolling a fair dice and observing the number that is rolled.

    (a) $P(A \cup B) \geq P(AB)$.

    (b) $P(AB) \geq P(A \cup B)$.

    (c) $P(AB^c) + P(A^cB) \leq 1$.

13. A bag contains 30 balls numbered 1 through 30. Seven balls are selected at random, one at a time, with replacement. What is the probability that exactly four of the selected balls have prime numbers on them?

14. A fair coin (one that shows heads and tails each with probability 1/2) is tossed repeatedly until the first time that the same face comes up twice in a row. Let X be the random variable that counts the number of tosses needed until this happens.

    (a) What are the possible values that X can take?

    (b) Compute the mass function of X.

    (c) What is the probability that it will take more than 3 tosses to first see the same face coming up twice in a row?

    (d) Compute the expectation of X.