Math 30530 — Introduction to Probability

Quiz 7 – Wednesday November 9, 2011

NAME: _____ *Solutions* _____

1. Adult Rat Terriers have a height that is normally distributed with mean 11", variance 1. What is the probability that a randomly chosen Rat Terrier has height ≥ 12.5 inches?

   Solution: Let X be the height of a randomly chosen adult Rat Terrier. We have that X is normal with $\mu = 11$ and $\sigma^2 = 1$. So

   $$\begin{aligned} P(X \geq 12.5) &= P\left(\frac{X - 11}{1} \geq \frac{12.5 - 11}{1}\right) \\ &= P\left(Z \geq 1.5\right) \\ &= 1 - P\left(Z \leq 1.5\right) \\ &= 1 - .9332 \\ &= .0668. \end{aligned}$$

2. The sub-breed of Saddle-back Rat Terriers has been selectively bred to have a height that is normally distributed with mean 11" and variance σ^2. It is known that a randomly chosen Saddle-back Rat Terrier has a 5% probability of being taller than 12.5 inches. What is σ?

   Let X be the height of a randomly chosen adult Saddle-back Rat Terrier. We have that X is normal with $\mu = 11$, but with $\sigma^2 = 1$ unknown. We know that $P(X \geq 12.5) = .05$, which is the same as $P((X - 11)/\sigma \geq (12.5 - 11)/\sigma) = .05$ or $P(Z \geq 1.5/\sigma) = .05$. Since $P(Z \leq 1.645) = .95$, we have $P(Z \geq 1.645) = .05$. So we want σ to satisfy $1.5/\sigma = 1.645$, or $\sigma = .912$.