Math 30530 — Introduction to Probability

Quiz 2 – Wednesday September 7, 2011

NAME: *Solution*

1. In how many different ways can 7 people be seated in a row if

   (a) There is a group of 4 among the 7 who insist on being seated together (in any order); or

   Solution: 4 ways to choose the four seats in a row occupied by the four who want to sit together; 4! ways to seat those four in the four selected seats; 3! ways to seat the rest, for a total of $(4)(4!)(3!) = 576$.

   (b) Within the group of 4 who insist on being seated together, there are 2 who insist on being seated side-by-side.

   Solution: 4 ways to choose the four seats in a row occupied by the four who want to sit together; 3 ways to choose the two seats in a row within the four for the two who want to sit together; 2! ways to seat those two; 2! ways to seat the other two from among the four; 3! ways to seat the rest, for a total of $(4)(3)(2!)(2!)(3!) = 288$.

2. Show, either by an algebraic or a counting argument, that

$$k\binom{n}{k} = n\binom{n-1}{k-1}.$$

Solution: Algebraic argument first:

$$k\binom{n}{k} = k\frac{n!}{k!(n-k)!} = \frac{n!}{(k-1)!(n-k)!} = n\frac{(n-1)!}{(k-1)!(n-k)!} = n\binom{n-1}{k-1}.$$

Now a counting argument: the left hand side counts the number of ways of selecting k people from n to form a committee, and then select one of the k to be the chair. The right hand side counts the number of ways of selecting one person from n to be chair of a committee, and then $k-1$ people from the remaining $n-1$ to join him on the committee. So both sides count the same thing — the number of committees-with-chair of size k from a group of size n — and thus are equal.