CHAPTER 6 PROBLEMS

29) a) S_1 = week 1 sales

S_2 = week 2 sales

S_1 = Normal $(2200, 230^2)$

S_2 = Normal $(2200, 230^2)$

Assuming independence,

$S_1 + S_2$ = Normal $(4400, 2 \cdot 230^2)$

$$P(S_1 + S_2 > 5000) = P\left(Z > \frac{5000-4400}{\sqrt{2}\,230}\right)$$

$$= P(Z > 1.84) = .0329$$

b) For each particular week,

$$P(\text{Sales} > 2000) = P\left(Z > \frac{2000-2200}{230}\right)$$

$$= P(Z > -.87) = .8078$$

So $P(\geq 2$ of next 3 weeks, Sales $> 2000)$

$= P(\text{Binomial}(3, .8078) \geq 2)$

assuming independence

$= .9034$

30. Let X denote Jill's score and let Y be Jack's score. Also, let Z denote a standard normal random variable.

(a)
$$\begin{aligned} P\{Y > X\} &= P\{Y - X > 0\} \\ &\approx P\{Y - X > .5\} \\ &= P\left\{\frac{Y - X - (160-170)}{\sqrt{(20)^2 + (15)^2}} > \frac{.5 - (160-170)}{\sqrt{(20)^2 + (15)^2}}\right\} \\ &\approx P\{Z > .42\} \approx .3372 \end{aligned}$$

(b)
$$\begin{aligned} P\{X + Y > 350\} &= P\{X + Y > 350.5\} \\ &= P\left\{\frac{X + Y - 330}{\sqrt{(20)^2 + (15)^2}} > \frac{20.5}{\sqrt{(20)^2 + (15)^2}}\right\} \\ &\approx P\{Z > .82\} \approx .2061 \end{aligned}$$

31. Let X and Y denote, respectively, the number of males and females in the sample that never eat breakfast. Since

$$E[X] = 50.4,\ \text{Var}(X) = 37.6992,\ \ E[Y] = 47.2,\ \text{Var}(Y) = 36.0608$$

it follows from the normal approximation to the binomial that X is approximately distributed as a normal random variable with mean 50.4 and variance 37.6992, and that Y is approximately distributed as a normal random variable with mean 47.2 and variance 36.0608. Let Z be a standard normal random variable.

(a)
$$\begin{aligned} P\{X + Y \geq 110\} &= P\{X + Y \geq 109.5\} \\ &= P\left\{\frac{X + Y - 97.6}{\sqrt{73.76}} \geq \frac{109.5 - 97.6}{\sqrt{73.76}}\right\} \\ &\approx P\{Z > 1.3856\} \approx .0829 \end{aligned}$$

(b)
$$\begin{aligned} P\{Y \geq X\} &= P\{Y - X \geq -.5\} \\ &= P\left\{\frac{Y - X - (-3.2)}{\sqrt{73.76}} \geq \frac{-.5 - (-3.2)}{\sqrt{73.76}}\right\} \\ &\approx P\{Z \geq .3144\} \approx .3766 \end{aligned}$$

32. (a) e^{-2}

(b) $1 - e^{-2} - 2e^{-2} = 1 - 3e^{-2}$
The number of typographical errors on each page should approximately be Poisson distributed and the sum of independent Poisson random variables is also a Poisson random variable.

34. Use the distribution of the sum of independent geometric random variables to obtain the result: $4(.7)^{12} - 3(.6)^{12}$

45. $$f_{X_{(3)}}(x) = \frac{5!}{2!2!}\left[\int_0^x xe^{-x}dx\right]^2 xe^{-x}\left[\int_x^\infty xe^{-x}dx\right]^2$$
$$= 30(x+1)^2e^{-2x}xe^{-x}[1-e^{-x}(x+1)]^2$$

THEORETICAL EXERCISES

10. If we let X_i denote the time between the i^{th} and $(i+1)^{st}$ failure, $i = 0, \ldots, n-2$, then it follows from Exercise 9 that the X_i are independent exponentials with rate 2λ. Hence, $\sum_{i=0}^{n-2} X_i$ the amount of time the light can operate is gamma distributed with parameters $(n-1, 2\lambda)$.