Chapter 4

Problems

1. $P\{X=4\} = \dfrac{\binom{4}{2}}{\binom{14}{2}} = \dfrac{6}{91}$ $\qquad$ $P\{X=0\} = \dfrac{\binom{2}{2}}{\binom{14}{2}} = \dfrac{1}{91}$

$P\{X=2\} = \dfrac{\binom{4}{2}\binom{2}{1}}{\binom{14}{2}} = \dfrac{8}{91}$ $\qquad$ $P\{X=-1\} = \dfrac{\binom{8}{1}\binom{2}{1}}{\binom{14}{2}} = \dfrac{16}{91}$

$P\{X=1\} = \dfrac{\binom{4}{1}\binom{8}{1}}{\binom{14}{2}} = \dfrac{32}{91}$ $\qquad$ $P\{X=-2\} = \dfrac{\binom{8}{2}}{\binom{14}{2}} = \dfrac{28}{91}$

4. $P\{X=1\} = 1/2$, $P\{X=2\} = \dfrac{5}{10}\dfrac{5}{9} = \dfrac{5}{18}$, $P\{X=3\} = \dfrac{5}{10}\dfrac{4}{9}\dfrac{5}{8} = \dfrac{5}{36}$,
$P\{X=4\} = \dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{5}{7} = \dfrac{10}{168}$, $P\{X=5\} = \dfrac{5\cdot4\cdot3\cdot2}{10\cdot9\cdot8\cdot7}\dfrac{5}{6} = \dfrac{5}{252}$,
$P\{X=6\} = \dfrac{5\cdot4\cdot3\cdot2\cdot1}{10\cdot9\cdot8\cdot7\cdot6} = \dfrac{1}{252}$

7) a) 1, 2, 3, 4, 5, 6
b) 1, 2, 3, 4, 5, 6
c) 2 through 12
d) −5 through 5

8. (a) $p(6) = 1-(5/6)^2 = 11/36$, $p(5) = 2\ 1/6\ \ 4/6 + (1/6)^2 = 9/36$
$p(4) = 2\ 1/6\ \ 3/6 + (1/6)^2 = 7/36$, $p(3) = 2\ 1/6\ \ 2/6 + (1/6)^2 = 5/36$
$p(2) = 2\ 1/6\ \ 1/6 + (1/6)^2 = 3/36$, $p(1) = 1/36$

(d) $p(5) = 1/36$, $p(4) = 2/36$, $p(3) = 3/36$, $p(2) = 4/36$, $p(1) = 5/36$
$p(0) = 6/36$, $p(-j) = p(j)$, $j > 0$

11. (a) $P\{\text{divisible by 3}\} = \dfrac{333}{1000}$ $\quad P\{\text{divisible by 105}\} = \dfrac{9}{1000}$

$P\{\text{divisible by 7}\} = \dfrac{142}{1000}$

$P\{\text{divisible by 15}\} = \dfrac{66}{1000}$

In limiting cases, probabilities converge to 1/3, 1/7, 1/15, 1/10

(b) $P\{\mu(N) \neq 0\} = P\{N \text{ is not divisible by } p_i^2, i \geq 1\}$

$= \prod_i P\{N \text{ is not divisible by } p_i^2\}$

$= \prod_i (1 - 1/p_i^2) = 6/\pi^2$

13. $p(0) = P\{\text{no sale on first and no sale on second}\}$
$= (.7)(.4) = .28$

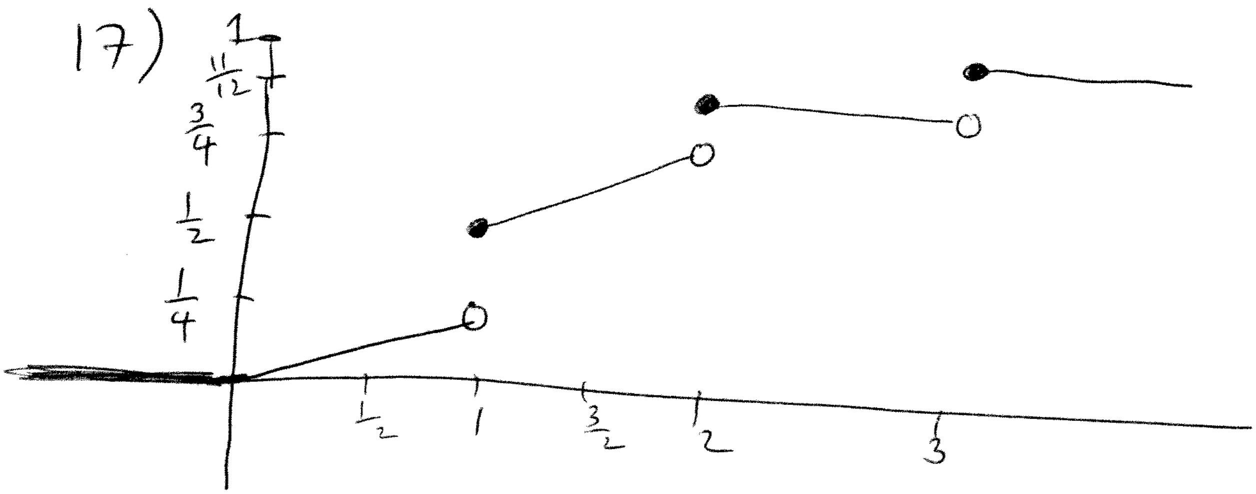
$p(500) = P\{1 \text{ sale and it is for standard}\}$
$= P\{1 \text{ sale}\}/2$
$= [P\{\text{sale, no sale}\} + P\{\text{no sale, sale}\}]/2$
$= [(.3)(.4) + (.7)(.6)]/2 = .27$

$p(1000) = P\{2 \text{ standard sales}\} + P\{1 \text{ sale for deluxe}\}$
$= (.3)(.6)(1/4) + P\{1 \text{ sale}\}/2$
$= .045 + .27 = .315$

$p(1500) = P\{2 \text{ sales, one deluxe and one standard}\}$
$= (.3)(.6)(1/2) = .09$

$p(2000) = P\{2 \text{ sales, both deluxe}\} = (.3)(.6)(1/4) = .045$

17)



a) $P(X=1) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$P(X=2) = \frac{11}{12} - \frac{3}{4} = \frac{1}{6}$

$P(X=3) = 1 - \frac{11}{12} = \frac{1}{12}$

b) $P(\frac{1}{2} < X < \frac{3}{2})$

$= F(\frac{3}{2}) - F(\frac{1}{2})$

$= \frac{5}{8} - \frac{1}{8} = \frac{1}{2}$

20. (a) $P\{x > 0\} = P\{\text{win first bet}\} + P\{\text{lose, win, win}\}$
$= 18/38 + (20/38)(18/38)^2 \approx .5918$

(b) No, because if the gambler wins then he or she wins \$1.
However, a loss would either be \$1 or \$3.

(c) $E[X] = 1[18/38 + (20/38)(18/38)^2] - [(20/38)2(20/38)(18/38)] - 3(20/38)^3 \approx -.108$

21. (a) $E[X]$ since whereas the bus driver selected is equally likely to be from any of the 4 buses, the student selected is more likely to have come from a bus carrying a large number of students.

27. $C - Ap = \dfrac{A}{10} \Rightarrow C = A\left(p + \dfrac{1}{10}\right)$

28. $3 \cdot \dfrac{4}{20} = 3/5$

29. If check 1, then (if desired) 2: Expected Cost $= C_1 + (1-p)C_2 + pR_1 + (1-p)R_2$;
if check 2, then 1: Expected Cost $= C_2 + pC_1 + pR_1 + (1-p)R_2$ so 1, 2, best if
$C_1 + (1-p)C_2 \le C_2 + pC_1$, or $C_1 \le \dfrac{p}{1-p}C_2$

30. $E[X] = \sum_{n=1}^{\infty} 2^n (1/2)^n = \infty$

(a) probably not

(b) yes, if you could play an arbitrarily large number of games

33) X = # papers he sells
$= \text{Binomial}\left(10, \frac{1}{3}\right)$

Suppose he buys k papers $(0 \le k \le 10)$

Cost : $10k$

Revenue : $\begin{cases} 15X & \text{if } X \le k \\ 15k & \text{if } X > k \end{cases}$

Profit : $\begin{cases} 15X - 10k & \text{if } X \le k \\ 5k & \text{if } X > k \end{cases}$

Profit = Profit(X)

$$E(\text{Profit}(X)) = \sum_{i=0}^{10} \text{Profit}(i) P(X=i)$$

$$= \sum_{i=0}^{k} (15i - 10k)\binom{10}{i}\left(\tfrac{1}{3}\right)^i \left(\tfrac{2}{3}\right)^{10-i}$$

$$+ \sum_{i=k+1}^{10} 5k \binom{10}{i}\left(\tfrac{1}{3}\right)^i \left(\tfrac{2}{3}\right)^{10-i}$$

Need to compute this for $k = 0, 1, \ldots, 10$, and see which gives largest value.

Via excel spreadsheet: Choose $k = 3$ (to get $E(\text{Profit}(X)) = 8.69$¢)

35. If X is the amount that you win, then

$P\{X = 1.10\} = 4/9 = 1 - P\{X = -1\}$
$E[X] = (1.1)4/9 - 5/9 = -.6/9 \approx =-.067$
$\text{Var}(X) = (1.1)^2(4/9) + 5/9 - (.6/9)^2 \approx 1.089$

39. $\binom{4}{2}(1/2)^4 = 3/8$

40. $\binom{5}{4}(1/3)^4(2/3)^1 + (1/3)^5 = 11/243$

Theoretical Exercises

4.
$$\sum_{i=1}^{\infty} P\{N \ge i\} = \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} P\{N = k\}$$
$$= \sum_{k=1}^{\infty}\sum_{i=1}^{\infty} P\{N = K\}$$
$$= \sum_{k=1}^{\infty} kP\{N = k\} = E[N].$$