Chapter 2

Problems

1. (a) $S = \{(r, r), (r, g), (r, b), (g, r), (g, g), (g, b), (b, r), b, g), (b, b)\}$
   (b) $S = \{(r, g), (r, b), (g, r), (g, b), (b, r), (b, g)\}$

2. $S = \{(n, x_1, \ldots, x_{n-1}), n \geq 1, x_i \neq 6, i = 1, \ldots, n-1\}$, with the interpretation that the outcome is $(n, x_1, \ldots, x_{n-1})$ if the first 6 appears on roll n, and x_i appears on roll, i, $i = 1, \ldots, n - 1$. The event $(\cup_{n=1}^{\infty} E_n)^c$ is the event that 6 never appears.

5. (a) $2^5 = 32$
   (b)
   $W = \{(1, 1, 1, 1, 1), (1, 1, 1, 1, 0), (1, 1, 1, 0, 1), (1, 1, 0, 1, 1), (1, 1, 1, 0, 0), (1, 1, 0, 1, 0)$
   $(1, 1, 0, 0, 1), (1, 1, 0, 0, 0), (1, 0, 1, 1, 1), (0, 1, 1, 1, 1), (1, 0, 1, 1, 0), (0, 1, 1, 1, 0), (0, 0, 1, 1, 1)$
   $(0, 0, 1, 1, 0), (1, 0, 1, 0, 1)\}$

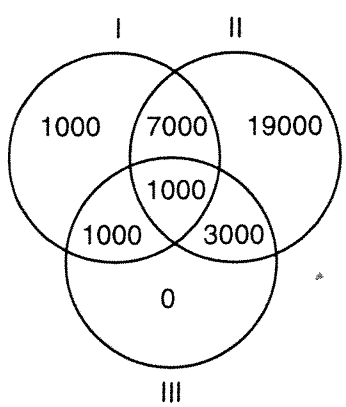
8. (a) .8
   (b) .3
   (c) 0

11. Let A be the event that a randomly chosen person is a cigarette smoker and let B be the event that she or he is a cigar smoker.

    (a) $1 - P(A \cup B) = 1 - (.07 + .28 - .05) = .7$. Hence, 70 percent smoke neither.

    (b) $P(A^cB) = P(B) - P(AB) = .07 - .05 = .02$. Hence, 2 percent smoke cigars but not cigarettes.

13.



(a) 20,000
(b) 12,000
(c) 11,000
(d) 68,000
(e) 10,000

17. $$\frac{\prod_{i=1}^{8} i^2}{64 \cdot 63 \cdots 58}$$

19. 4/36 + 4/36 +1/36 + 1/36 = 5/18

23. The answer is 5/12, which can be seen as follows:

$$\begin{aligned} 1 &= P\{\text{first higher}\} + P\{\text{second higher}\} + p\{\text{same}\} \\ &= 2P\{\text{second higher}\} + p\{\text{same}\} \\ &= 2P\{\text{second higher}\} + 1/6 \end{aligned}$$

Another way of solving is to list all the outcomes for which the second is higher. There is 1 outcome when the second die lands on two, 2 when it lands on three, 3 when it lands on four, 4 when it lands on five, and 5 when it lands on six. Hence, the probability is $(1 + 2 + 3 + 4 + 5)/36 = 5/12$.

28. $$P\{\text{same}\} = \frac{\binom{5}{3}+\binom{6}{3}+\binom{8}{3}}{\binom{19}{3}}$$

$$P\{\text{different}\} = \binom{5}{1}\binom{6}{1}\binom{8}{1}\Big/\binom{19}{3}$$

If sampling is with replacement

$$P\{\text{same}\} = \frac{5^3+6^3+8^3}{(19)^3}$$

$$\begin{aligned} P\{\text{different}\} &= P(RBG) + P\{BRG) + P(\text{RGB}) + \ldots + P(GBR) \\ &= \frac{6\cdot 5\cdot 6\cdot 8}{(19)^3} \end{aligned}$$

29. (a) $$\frac{n(n-1)+m(m-1)}{(n+m)(n+m-1)}$$

(b) Putting all terms over the common denominator $(n + m)^2(n + m - 1)$ shows that we must prove that

$$n^2(n + m - 1) + m^2(n + m - 1) \geq n(n - 1)(n + m) + m(m - 1)(n + m)$$

which is immediate upon multiplying through and simplifying.

30. (a) $$\frac{\binom{7}{3}\binom{8}{3}3!}{\binom{8}{4}\binom{9}{4}4!} = 1/18$$

32. $$\frac{g(b+g-1)!}{(b+g)!} = \frac{g}{b+g}$$

34. $$\binom{32}{13}\Big/\binom{52}{13}$$

35. (a) $\dfrac{\binom{12}{3}\binom{16}{2}\binom{18}{2}}{\binom{46}{7}}$

(b) $1-\dfrac{\binom{34}{7}}{\binom{46}{7}}-\dfrac{\binom{12}{1}\binom{34}{6}}{\binom{46}{7}}$

(c) $\dfrac{\binom{12}{7}+\binom{16}{7}+\binom{18}{7}}{\binom{46}{7}}$

(d) $P(R_3 \cup B_3) = P(R_3) + P(B_3) - P(R_3B_3) = \dfrac{\binom{12}{3}\binom{34}{4}}{\binom{46}{7}} + \dfrac{\binom{16}{3}\binom{30}{4}}{\binom{46}{7}} - \dfrac{\binom{12}{3}\binom{16}{3}\binom{18}{1}}{\binom{46}{7}}$

39. $\dfrac{5 \cdot 4 \cdot 3}{5 \cdot 5 \cdot 5} = \dfrac{12}{25}$

40. $P\{1\} = \dfrac{4}{44} = \dfrac{1}{64}$

$P\{2\} = \binom{4}{2}\left[4 + \binom{4}{2} + 4\right] \Big/ 4^4 = \dfrac{84}{256}$

$P\{3\} = \binom{4}{3}\binom{3}{1}\dfrac{4!}{2!} \Big/ 4^4 = \dfrac{36}{64}$

$P\{4\} = \dfrac{4!}{4^4} = \dfrac{6}{64}$

43. $\dfrac{2(n-1)(n-2)}{n!} = \dfrac{2}{n}$ in a line

$\dfrac{2n(n-2)!}{n!} = \dfrac{2}{n-1}$ if in a circle, $n \geq 2$

46. If n in the room,

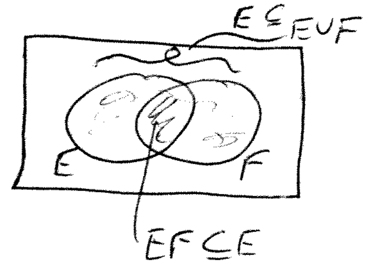
$$P\{\text{all different}\} = \frac{12 \cdot 11 \cdot \quad \cdot (13-n)}{12 \cdot 12 \cdot \quad \cdot 12}$$

When $n = 5$ this falls below 1/2. (Its value when $n = 5$ is .3819)

Theoretical Exercises

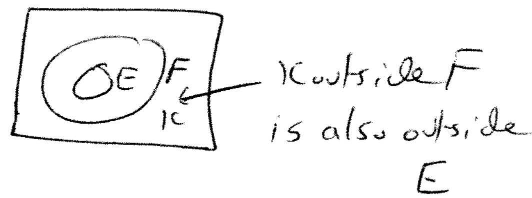
1) $E = EF \cup EF^c$, so $EF \subseteq E$

$E \cup F = E \cup EF^c$, so $E \subseteq E \cup F$



2) If $x \in F^c$ then $x \notin F$, so $x \notin E$, so $x \in E^c$

↑ Because $E \subset F$



6. (a) EF^cG^c

(b) EF^cG

(c) $E \cup F \cup G$

(d) $EF \cup EG \cup FG$

(e) EFG

(f) $E^cF^cG^c$

(g) $E^cF^cG^c \cup EF^cG^c \cup E^cFG^c \cup E^cF^cG$

(h) $(EFG)^c$

(i) $EFG^c \cup EF^cG \cup E^cFG$

(j) S

11. $1 \geq P(E \cup F) = P(E) + P(F) - P(EF)$

12. $P(EF^c \cup E^cF) = P(EF^c) + P(E^cF)$
$= P(E) - P(EF) + P(F) - P(EF)$

15. $\dfrac{\binom{M}{k}\binom{N}{r-k}}{\binom{M+N}{r}}$

18) Definitely $f_2 = 2$ (H or T)

and $f_3 = 3$ (HT, TH or TT)

For $n \geq 3$: If a string begins with T, then it can be followed by any string of length $n-1$ that has no two consecutive H's; this gives f_{n-1} strings.

If a string begins H, then it must be followed by T, and then any string of length $n-2$ that has no two consecutive H's; this gives f_{n-2} strings.

So $f_n = f_{n-1} + f_{n-2}$ (*)

If we let $f_0 = 1$ and $f_1 = 2$, then the recurrence (*) gives $f_3 = 3$, so we can start at f_0

$$P_n = \frac{f_n}{2^n}$$

f_n ← # successful outcomes

2^n ← # outcomes

Using the recurrence, get $f_{10} = 144$,

So $P_{10} = \frac{144}{2^{10}} = \frac{9}{64}$.