MATH 30530, Fall 2009 – Homework 8 Solutions

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3. **(a)** $1/(1/12) = 12$. **(b)** $\left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right) \approx 0.07$.

5. $\binom{7}{2}(0.2)^3(0.8)^5 \approx 0.055$.

7. $\frac{\binom{5}{1}\binom{45}{7}}{\binom{50}{8}} = .42\ldots$

9. We have

$$\frac{P(N = n)}{P(X = x)} = \frac{\binom{n-1}{x-1}p^x(1-p)^{n-x}}{\binom{n}{x}p^x(1-p)^{n-x}} = \frac{x}{n}.$$

12. The probability that a random bridge hand has three aces is

$$p = \frac{\binom{4}{3}\binom{48}{10}}{\binom{52}{13}} = 0.0412.$$

Therefore, the average number of bridge hands until one has three aces is $1/p = 1/0.0412 = 24.27$.

15. Let X be the number of good diskettes in the sample. The desired probability is

$$P(X \geq 9) = P(X = 9) + P(X = 10) = \frac{\binom{10}{1}\binom{90}{9}}{\binom{100}{10}} + \frac{\binom{90}{10}\binom{10}{0}}{\binom{100}{10}} \approx 0.74.$$

17. The transmission of a message takes more than t minutes, if the first $[t/2] + 1$ times it is sent it will be garbled, where $[t/2]$ is the greatest integer less than or equal to $t/2$. The probability of this is $p^{[t/2]+1}$.

22. **(a)** Since

$$\frac{P(X = n-1)}{P(X = n)} = \frac{1}{1-p} > 1,$$

$P(X = n)$ is a decreasing function of n; hence its maximum is at $n = 1$.

(b) The probability that X is even is given by

$$\sum_{k=1}^{\infty} P(X = 2k) = \sum_{k=1}^{\infty} p(1-p)^{2k-1} = \frac{p(1-p)}{1-(1-p)^2} = \frac{1-p}{2-p}.$$

~~(c) We want to show the following:~~

27. Let X be the number of rolls until Adam gets a six. Let Y be the number of rolls of the die until Andrew rolls an odd number. Since the events $(X = i)$, $1 \le i < \infty$, form a partition of the sample space, by Theorem 3.4,

$$P(Y > X) = \sum_{i=1}^{\infty} P(Y > X \mid X = i)P(X = i) = \sum_{i=1}^{\infty} P(Y > i)P(X = i)$$

$$= \sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^i \cdot \left(\frac{5}{6}\right)^{i-1} \frac{1}{6} = \frac{6}{5} \cdot \frac{1}{6} \sum_{i=1}^{\infty} \left(\frac{5}{12}\right)^i = \frac{1}{5} \cdot \frac{\frac{5}{12}}{1 - \frac{5}{12}} = \frac{1}{7},$$

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1. (a) $\int_0^{\infty} ce^{-3x}\,dx = 1 \Longrightarrow c = 3.$

(b) $P(0 < X \le 1/2) = \int_0^{1/2} 3e^{-3x}\,dx = 1 - e^{-3/2} \approx 0.78.$

3. (a) $\int_1^2 c(x-1)(2-x)\,dx = 1 \Longrightarrow c\Big[-\frac{x^3}{3} + \frac{3x^2}{2} - 2x\Big]_1^2 = 1 \Longrightarrow c = 6.$

(b) $F(x) = \int_1^x 6(x-1)(2-x)\,dx, \quad 1 \le x < 2.$ Thus

$$F(x) = \begin{cases} 0 & x < 1 \\ -2x^3 + 9x^2 - 12x + 5 & 1 \le x < 2 \\ 1 & x \ge 2. \end{cases}$$

(c) $P(X < 5/4) = F(5/4) = 5/32,$
$P(3/2 \le X \le 2) = F(2) - F(3/2) = 1 - (1/2) = 1/2.$

7. (a) Let F be the distribution function of X. Then X is symmetric about α if and only if for all x, $1 - F(\alpha + x) = F(\alpha - x)$, or upon differentiation $f(\alpha + x) = f(\alpha - x)$.
(b) $f(\alpha + x) = f(\alpha - x)$ if and only if $(\alpha - x - 3)^2 = (\alpha + x - 3)^2$. This is true for all x, if and only if $\alpha - x - 3 = -(\alpha + x - 3)$ which gives $\alpha = 3$. A similar argument shows that g is symmetric about $\alpha = 1$.

9. $P(X > 15) = \int_{15}^{\infty} \frac{1}{15} e^{-x/15}\,dx = \frac{1}{e}$. Thus the answer is

$$\sum_{i=4}^{8} \binom{8}{i} \left(\frac{1}{e}\right)^i \left(1 - \frac{1}{e}\right)^{8-i} = 0.3327.$$