MATH 30440, SPRING 2010 – HOMEWORK 9 SOLS

7.32) a) $X_{n+1} = \text{Normal}(\mu, \sigma^2)$

$\overline{X_n} = \text{Normal}\left(\mu, \frac{\sigma^2}{n}\right)$

$$X_{n+1} - \overline{X_n} = \text{Normal}\left(0, \left(1+\tfrac{1}{n}\right)\sigma^2\right)$$

b) $$\frac{X_{n+1} - \overline{X_n}}{S_n\sqrt{1+\frac{1}{n}}} = \underbrace{\frac{X_{n+1} - \overline{X_n}}{\sigma\sqrt{1+\frac{1}{n}}}}_{=Z} \times \frac{S_n}{\sigma}$$

$$= \frac{Z}{\sqrt{\frac{(n-1)S_n^2}{\sigma^2(n-1)}}}$$

$$= \frac{Z}{\sqrt{\frac{\chi^2_{df=n-1}}{n-1}}}$$

$$= t_{n-1}.$$

c) with prob $1-\alpha$,

$$-t_{n-1,\frac{\alpha}{2}} \leq \frac{X_{n+1} - \overline{X_n}}{S_n\sqrt{1+\frac{1}{n}}} \leq t_{n-1,\frac{\alpha}{2}}$$

so $\overline{X_n} - t_{n-1,\frac{\alpha}{2}} S_n\sqrt{1+\frac{1}{n}} \leq X_{n+1} \leq \overline{X_n} + t_{n-1,\frac{\alpha}{2}} S_n\sqrt{1+\frac{1}{n}}$

$100(1-\alpha)\%$ confidence interval:

$$\overline{X_n} \pm t_{n-1,\frac{\alpha}{2}} S_n \sqrt{1+\frac{1}{n}}$$

d) 100(1−α)% of the time that we perform sampling n+1 times, and construct the interval $\overline{X_n} \pm$ etc., X_{n+1} will lie in this interval

7.34) Want value v s.t with 90% confidence, $v \geq \mu$, ie $\mu \leq v$.

So we want v to be the ~~left~~ right hand endpoint of a 90% lower confidence interval

$$v = \overline{X} + t_{29,.1} \frac{5}{\sqrt{30}}$$

$$= 2.5 + 1.311 \times \frac{2.12}{\sqrt{30}}$$

7.36) a) From excel, $s^2 = 32.33$

b) 99% of time,

$$\chi^2_{9,.995} \le \frac{9s^2}{\sigma^2} \le \chi^2_{9,.005}$$

So

$$\frac{9s^2}{\chi^2_{9,.005}} \le \sigma^2 \le \frac{9s^2}{\chi^2_{9,.995}}$$

99% confidence interval:

$$\left(\frac{9 \times 32.33}{23.589}, \frac{9 \times 32.33}{1.735} \right)$$

c) 90% of time, $\chi^2_{9,.9} \le \frac{9s^2}{\sigma^2}$

So

$$\sigma^2 \le \frac{9s^2}{\chi^2_{9,.9}} = \frac{9 \times 32.33}{4.2}$$

7.38) $S_1^2 = 2.5$ (first 5 samples)

$S_2^2 = 7$ (last 3 samples)

Pooled estimator:

$$S_p^2 = \frac{4S_1^2 + 2S_2^2}{6} = 4$$

7.39) a) From excel, $S^2 = 6.48 \times 10^{-5}$

$S = .008$

b) 90% interval for σ^2:

$$\left(\frac{7 \times 6.48 \times 10^{-5}}{15.507}, \frac{7 \times 6.48 \times 10^{-5}}{2.733} \right)$$

90% interval for σ:

$$\left(\sqrt{\frac{7 \times 6.48 \times 10^{-5}}{15.507}}, \sqrt{\frac{7 \times 6.48 \times 10^{-5}}{2.733}} \right)$$

7.41) Use excel or calculator to calculate

$\overline{X_1}$ (sample mean from type I)

$\overline{X_2}$ (" " " " II)

S_1^2 (sample variance from type I)

S_2^2 (" " " " II)

Then calculate pooled estimator for common variance:

$$S_p^2 = \frac{9S_1^2 + 9S_2^2}{18}$$

a) $\left(\overline{X_1} - \overline{X_2}\right) \pm t_{18,.025}\sqrt{\frac{S_p^2}{10} + \frac{S_p^2}{10}}$

b) $\left(\overline{X_1} - \overline{X_2} - t_{18,.05}\sqrt{\frac{S_p^2}{10} + \frac{S_p^2}{10}}\ ,\ \infty\right)$

c) $\left(-\infty,\ \overline{X_1} - \overline{X_2} + t_{18,.05}\sqrt{\frac{S_p^2}{10} + \frac{S_p^2}{10}}\right)$

7.42) $\overline{X_1} = 120 \qquad S_1^2 = 4 \qquad n = 36$

$\overline{X_2} = 130 \qquad S_2^2 = 5 \qquad m = 64$

$$S_p^2 = \frac{35\times 4 + 63\times 5}{108} = 4.642$$

99% con int for $\mu_1 - \mu_2$:

$$-10 \pm t_{98,.005}\sqrt{\frac{4.642}{36} + \frac{4.642}{64}}$$

$$t_{98,.005} = t_{\infty,.005}$$

7.43) 99% con int for $\mu_1 - \mu_2$:

$$-10 \pm z_{.005}\sqrt{\frac{4}{36} + \frac{5}{64}}$$

7.49) $\hat{p} = .5106$

a) 90% Con int :

either $\hat{p} \pm \underset{\substack{\uparrow \\ 1.645}}{z_{.05}} \sqrt{\dfrac{\hat{p}(1-\hat{p})}{10000}}$ (using $\hat{p}$ in estimate for variance)

or $\hat{p} \pm 1.645 \sqrt{\dfrac{\frac{1}{2} \times \frac{1}{2}}{10000}}$ (using worst-case estimate $p = \frac{1}{2}$)

b) Same with 1.645 replaced by 2.58

7.50) 90% Con int is $\hat{p} \pm 1.645 \sqrt{\dfrac{\frac{1}{2} \times \frac{1}{2}}{n}}$

(use worst-case estimate $p = \frac{1}{2}$ since we don't know any estimate for $\hat{p}$)

Choose n so that $1.645 \sqrt{\dfrac{\frac{1}{2} \times \frac{1}{2}}{n}} = .02$

7.54) $\hat{p} = .17$

95% con int for proportion defective:

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{100}} \quad \text{or} \quad \hat{p} \pm 1.96\sqrt{\frac{\frac{1}{2}\times\frac{1}{2}}{100}}$$

99% upper confidence interval:

$$\left(\hat{p} - 2.33\sqrt{\frac{\hat{p}(1-\hat{p})}{100}},\ 1\right)$$

$$\left[\text{or } \left(\hat{p} - 2.33\sqrt{\frac{\frac{1}{2}\times\frac{1}{2}}{100}},\ 1\right)\right]$$

7.55) a) $\hat{p} = .67$

b) 95% confidence error is

$$\pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \text{or} \quad \pm 1.96\sqrt{\frac{\frac{1}{2}\times\frac{1}{2}}{n}}$$

Choose n large enough that

$1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $\left(\text{or } 1.96\sqrt{\frac{\frac{1}{2}\times\frac{1}{2}}{n}}\right)$ is $.02$.