MATH 30440 - SPRING 2010 - HOMEWORK 8 SOLS

7.1) Suppose readings are $x_1, \dots, x_n$.

Joint density at these readings is

$$f(\theta) = \begin{cases} e^{-(x_1-\theta)} e^{-(x_2-\theta)} \cdots e^{-(x_n-\theta)} & \text{if all } x_i \geq \theta \\ 0 & \text{o/wise} \end{cases}$$

In interesting (non-zero) region,

$$f(\theta) = e^{-\sum x_i + n\theta}$$

$$\log f(\theta) = -\sum x_i + n\theta$$

Derivative $= n$; so $\log f(\theta)$ always increasing.

We make it as large as possible by choosing θ as large as possible.

Since we have to have $x_i \geq \theta \ \forall i$,

Choose θ = smallest number among $x_1, \dots, x_n$.

7.3) Joint density of n independent copies of Normal (μ, σ^2) at readings $x_1, \ldots, x_n$

is

$$\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^n e^{-\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}}$$

Log of this is $-n \log \sqrt{2\pi} - n \log \sigma - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i-\mu)^2$

Derivative: $\frac{-n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^{n} (x_i-\mu)^2$

$$= 0 \text{ when } \sigma^2 = \frac{\sum_{i=1}^{n} (x_i-\mu)^2}{n} \leftarrow \text{MLE}$$

$$\overline{E(\text{MLE})} = E\left(\frac{\sum_{i=1}^{n} (x_i-\mu)^2}{n}\right) = \frac{1}{n} \sum_{i=1}^{n} E\left((x_i-\mu)^2\right)$$

$\|\leftarrow$ definition!

$$= \frac{1}{n} \sum_{i=1}^{n} Var(X_i)$$

$$= \frac{1}{n} n \sigma^2$$

$$= \sigma^2$$

7.7) Best thing we can do, I think, is use

$$\bar{X} = .254$$
$$S = .005$$

as estimates for μ and σ, and calculate

$$P\left(\text{Normal}\left(.254, (.005)^2\right) > .260\right)$$
$$+ P\left(\text{"} \quad (\text{"}, \text{"}) < .240\right)$$

ie $P(Z > 1.2) + P(Z < -2.8)$.

No really good way to work in fact that there are 20 samples.

7.8) $\bar{X} = 3.1502$

95% con int: $\bar{X} \pm 1.96 \frac{.1}{\sqrt{5}}$

99% con int $\bar{X} \pm 2.58 \frac{.1}{\sqrt{5}}$

7.9) $\overline{X} = 11.48$

a) 95% con int : $\overline{X} \pm 1.96 \frac{.08}{\sqrt{10}}$

b) 95% lower : $\left(-\infty,\ \overline{X} + 1.645 \frac{.08}{\sqrt{10}}\right)$

c) 95% upper : $\left(\overline{X} - 1.645 \frac{.08}{\sqrt{10}},\ \infty\right)$

7.11) a) $X_{n+1} = \text{Normal}(\mu, 1)$

$\overline{X_n} = \text{Normal}(\mu, \frac{1}{n})$

$X_{n+1} - \overline{X_n} = \text{Normal}(0, 1+\frac{1}{n})$

b) 90% of time,

$$-1.645 \le \underbrace{\frac{X_{n+1} - \overline{X_n}}{\sqrt{1+\frac{1}{n}}}}_{= Z} \le 1.645$$

so $\overline{X_n} - 1.645\sqrt{1+\frac{1}{n}} \le X_{n+1} \le \overline{X_n} + 1.645\sqrt{1+\frac{1}{n}}$

Here $\overline{X} = 4$; so 90% con int for X_{n+1} is

$$\left(4 - 1.645\sqrt{1+\frac{1}{n}},\ 4 + 1.645\sqrt{1+\frac{1}{n}}\right)$$

7.15) Want $\mu \leq c$ with 99% confidence

So want <u>lower</u> confidence interval

$$\left(-\infty,\ \overline{X} + t_{19,.01}\frac{s}{\sqrt{20}}\right)$$

So $c = \overline{X} + t_{19,.01}\frac{s}{\sqrt{20}}$

$= 1.2 + 2.539\frac{\sqrt{.04}}{\sqrt{20}}$

7.14) $\overline{X} \pm t_{19,.005}\frac{s}{\sqrt{n}}$ or

$1.2 \pm 2.861\frac{\sqrt{.04}}{\sqrt{20}}$

7.17) $\left.\begin{aligned}\overline{X} &= 333.58\\ s^2 &= 55.18\end{aligned}\right\}$ calculated with excel

95% confidence int: $\overline{X} \pm t_{23,.025}\frac{s}{\sqrt{24}}$

99% confidence int: $\overline{X} \pm t_{23,.005}\frac{s}{\sqrt{24}}$

7.21) 95% Con int : $\overline{X} \pm t_{99,.025} \frac{16}{\sqrt{100}}$

$= 320 \pm 1.96 \times 1.6$

7.26) $\left.\begin{array}{l} \overline{X} = 2062.75 \\ s^2 = 1088757 \end{array}\right\}$ Calculated from excel

a) 95% Con. int : $\overline{X} \pm t_{19,.025} \frac{s}{\sqrt{20}}$

b) 99% Con int : $\overline{X} \pm t_{19,.005} \frac{s}{\sqrt{20}}$

c) Want v so that $v \leq \mu$. So v will be the left endpoint of a one-sided upper confidence interval

$v = \overline{X} - t_{19,.05} \frac{s}{\sqrt{20}}$.