MATH 30440, SPRING 2010 - HOMEWORK 7 SOLS

6.1) a) $(n=2)$ 9 possible values for pair (X_1, X_2):

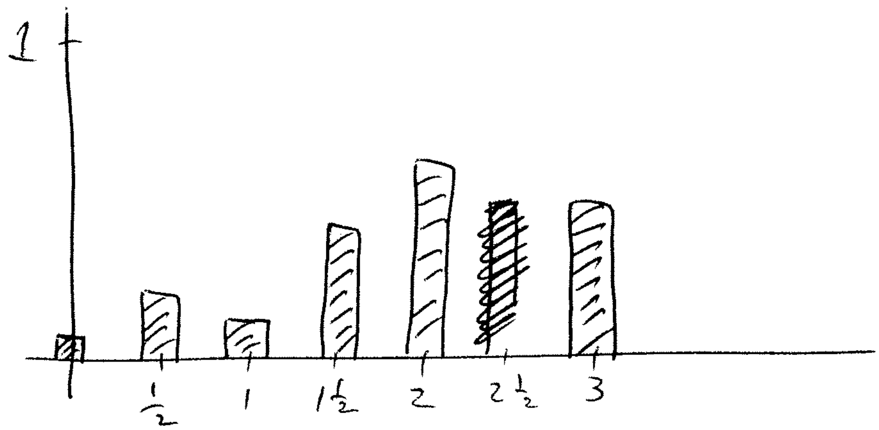
Cases:

	X_1	X_2	$\bar{X}$	prob
i)	$X_1=0$	$X_2=0$	$\bar{X}=0$	.04 ✓
ii)	0	1	$\frac{1}{2}$	.06 ✓
iii)	0	3	$1\frac{1}{2}$	.1 ✓
iv)	1	0	$\frac{1}{2}$	.06 ✓
v)	1	1	1	.09 ✓
vi)	1	3	2	.15 ✓
vii)	3	0	$1\frac{1}{2}$	.1 ✓
viii)	3	1	2	.15 ✓
ix)	3	3	3	.25 ✓

So
$$\bar{X} = \begin{cases} 0 & \text{w/ prob } .04 \\ \frac{1}{2} & \text{w/ prob } .12 \\ 1 & \text{w/ prob } .09 \\ 1\frac{1}{2} & \text{w/ prob } .2 \\ 2 & \text{w/ prob } .3 \\ 3 & \text{w/ prob } .25 \end{cases}$$

Check: these should add to 1

Plot of mass function:



$E(\bar{X}) = 1.8 \quad (= E(X))$

$Var(\bar{X}) = .78 \quad (= Var(X)/2)$

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b) Similarly, except mass function is more concentrated around 1.8

$E(\bar{X}) = 1.8$

$Var(\bar{X}) = .52$

6.4) X = net result of a single roll

$$X = \begin{cases} +35 & \bar{w} \text{ prob } \frac{1}{38} \\ -1 & \bar{w} \text{ prob } \frac{37}{38} \end{cases}$$

$E(X) = -\frac{1}{19}$ $\qquad$ $Var(X) = 33.2$

If you play n times, your winning is

$$\underset{\substack{\uparrow \\ \text{CLT}}}{X_1 + \dots + X_n \approx} \text{Normal}\left(\frac{-n}{19}, 33.2n\right)$$

So we want $P\left(\text{Normal}\left(\frac{-n}{19}, 33.2n\right) > 0\right)$

$$= P\left(Z > \frac{\frac{n}{19}}{\sqrt{33.2n}}\right)$$

$$= P\left(Z > \frac{\sqrt{n}}{109}\right)$$

a) $n = 34$: $P(Z > .05..)$

b) $n = 1000$: $P(Z > .29...)$

c) $n = 100\,000$: $P(Z > 2.9...)$

6.5) Daily amount of snow :

Normal $(1.5, (.3)^2)$

Amount in 50 days : Normal $(75, 4.5)$

$\uparrow$ 1.5×50 $\quad$ $\uparrow$ $(.3)^2 \times 50$

a) $P(\text{enough salt}) = P(\text{Normal}(75, 4.5) \leq 80)$

$= P(Z \leq 2.35\ldots)$

b) Assumption : Snowfall from day to day independent

c) Probably not justified!

6.8) $P(13 \text{ or more needed})$

$= P(\text{First twelve last} < 1 \text{year})$

$= P(\text{Normal}(12 \times 5, (1.5)^2 \times 5) < 52)$

$= P(Z \leq -2.38\ldots)$

6.10) If claim is true,

$$\bar{X} = \text{sample mean} = \text{Normal}\left(2.2, \frac{(.3)^2}{100}\right)$$

$$P\left(\text{Normal}\left(2.2, \frac{(.3)^2}{100}\right) \geq 3.1\right)$$

$$= P(Z \geq 30) \quad (= 0)$$

6.12) a) Average test score for class of size 25 is

$$\bar{X} \approx \text{Normal}\left(77, \frac{(15)^2}{25}\right)$$

$$P(72 \leq \bar{X} \leq 82)$$

$$= P(72 \leq \text{Normal}(77, 9) \leq 82)$$

$$= P(-1.66 \leq Z \leq +1.66)$$

b) Average for class of size 64 is

$$\bar{X} \approx \text{Normal}\left(77, \frac{(15)^2}{64}\right)$$

$$P(72 \leq \bar{X} \leq 82) = P(-2.66 \leq Z \leq 2.66)$$

[Much higher than part a)]

c) Difference in averages is

$$\text{Normal}\left(77-77,\ \frac{(15)^2}{25}+\frac{(15)^2}{64}\right)$$

$$= \text{Normal}\ (0,\ 12.51\ldots)$$

$$P(\text{Difference} > 0) = P(\text{Normal}\ (0,\ 12.51) > 0)$$

$$= P(Z > 0)$$

$$= .5$$

d) Smaller class is more likely to have average away from 77 (More Variance in average of smaller number of readings)

6.14) $X = \#$ defective in 1000

$= \text{Binomial}(1000, .25)$

$\approx \text{Normal}(250, 187.5)$

$\sigma \approx 13.7$

$P(X < 200) = P(\text{Normal}(250, 187.5) < 200)$

$= P(Z < -3.65) \quad (\approx 0)$

6.15) a) No, success probabilities are not the same for each trial

b) $X_A = \text{Binomial}(32, .5)$

$X_B = \text{Binomial}(28, .7)$

c) $X = X_A + X_B$

d) $X_A \approx \text{Normal}(16, 8)$

$X_B \approx \text{Normal}(19.6, 5.88)$

$X \approx \text{Normal}(35.6, 13.88)$

$P(X \geq 40) = P(\text{Normal}(35.6, 13.88) \geq 40)$

$= P(Z \geq 1.18\ldots)$

6.18) $\frac{4S^2}{\sigma^2} = \chi^2_{df=4}$ (Since we're testing 5 times, $n-1=4$)

a) $P\left(\frac{S^2}{\sigma^2} \leq 1.8\right) = P\left(\frac{4S^2}{\sigma^2} \leq 7.2\right)$

$= P(\chi^2_{df=4} \leq 7.2)$

b) $P\left(.85 \leq \frac{S^2}{\sigma^2} \leq 1.15\right)$

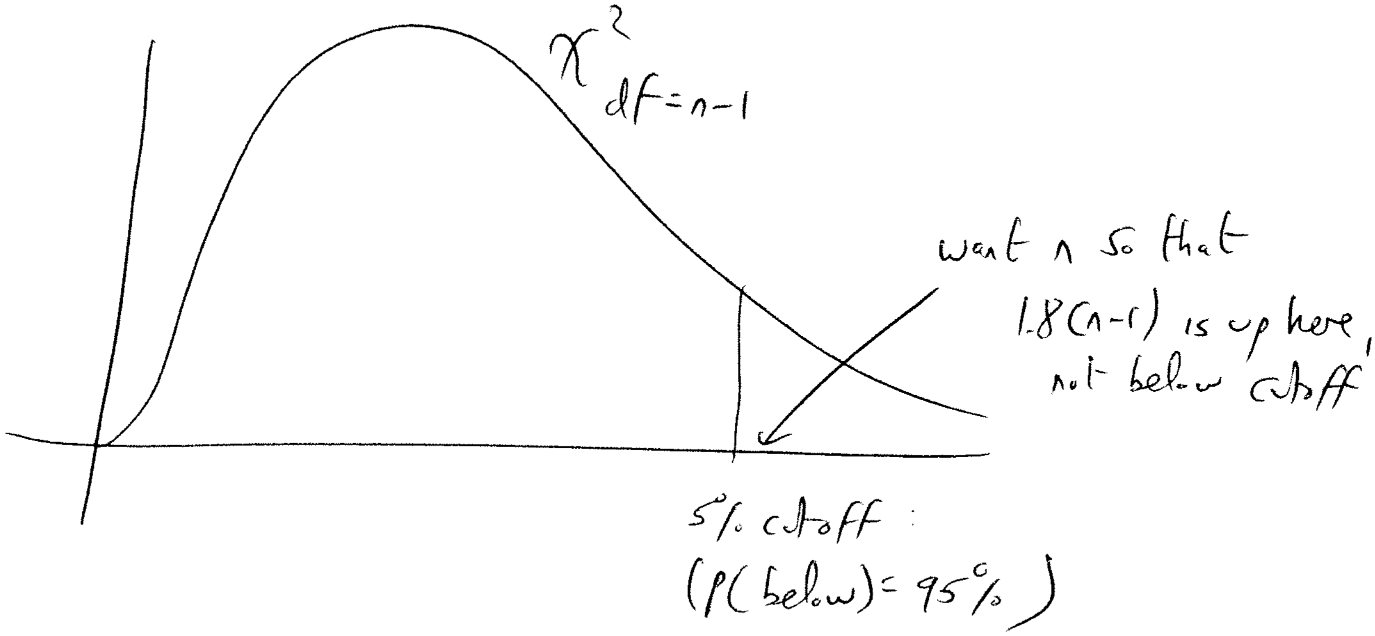
$= P\left(3.4 \leq \frac{4S^2}{\sigma^2} = \chi^2_{df=4} \leq 4.6\right)$

6.19) Sample size n

$$P\left(\frac{S^2}{\sigma^2} \le 1.8\right) = P\left(\frac{(n-1)S^2}{\sigma^2} \le (n-1)1.8\right)$$

$$= P\left(\chi^2_{df=n-1} \le 1.8(n-1)\right)$$

Use table to check which is the first value of n for which $1.8(n-1)$ exceeds the .05 cutoff value for $\chi^2_{df=n-1}$



6.22) For random sample of size n,

$X = \#$ in favour

$= \text{Binomial}(n, .52)$

$\approx \text{Normal}(.52n, .2496n)$

So $P(X \geq \frac{n}{2}) \approx P(\text{Normal}(.52n, .2496n) \geq .5n)$

$= P(Z \geq -.04\sqrt{n})$

a) $P(Z \geq -.12\ldots)$

b) $P(Z \geq -.4)$

c) $P(Z \geq -1.26\ldots)$

d) $P(Z \geq -4)$ (≈ 1)

6.26) a) From table,

$P(\text{randomly chosen woman earned} < 20{,}000)$

$= .028 + .104 + .41 = .542$

So X = # women earning < 20,000 in random sample of 1000 women

$= \text{Binomial}(1000, .542)$

$\approx \text{Normal}(542, 248.24)$

So $P(X \geq 500) = P(\text{Normal}(542, 248.24) \geq 500)$

$= P(Z \geq -2.66\ldots)$

Remaining parts similar; for part c), since samples of men + women are (presumably) independent, multiply individual probabilities to get P(both happen)

6.28) $\bar{X}$ = Average of 144

$$\approx \text{Normal}\left(517, \frac{(120)^2}{144}\right)$$

a) $P(\bar{X} > 507) = P\left(\text{Normal}\left(517, \frac{(120)^2}{144}\right) > 507\right)$

$$= P\left(Z > \frac{507 - 517}{\sqrt{\frac{(120)^2}{144}}}\right)$$

$$= P(Z > -1)$$

b), c), d) similar.