MATH 30440, SEC 01, SPRING 2010

HOMEWORK 2 SOLUTIONS

3.18) a) # ways of placing so that a boy is in 4^{th} seat is $5 \times 14!$

(just fill the 4^{th} seat first).

So probability $= \dfrac{5 \times 14!}{15!} = \dfrac{1}{3}$

b) Same as a) : $\dfrac{1}{3}$

c) Now # good ways is $1 \times 14!$, so

probability $= \dfrac{1 \times 14!}{15!} = \dfrac{1}{15}$

3.21) a) $P(A) = .6$

$P(B|A^c) = .1$

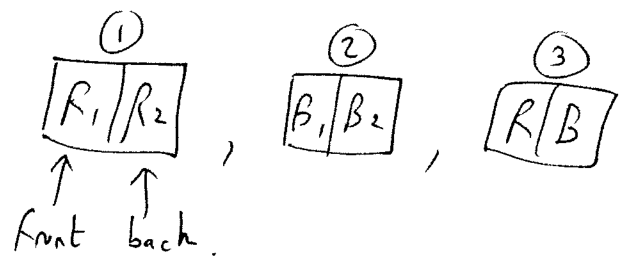
I'm assuming from wording that if A occurs, B can't, so A and B are mut. exc.

$$\underset{\substack{\uparrow \\ \text{at least one}}}{P(A \cup B)} = P(A) + P(B) = P(A) + \underbrace{P(B|A^c)P(A^c)}_{\substack{\text{Law of total prob;} \\ P(B|A)=0}}$$

$= .6 + .1 \times .4 = .64.$

b) $P(AB) = P(A)P(B)$, assuming that the events are independent.

3.23) Cards are



Experiment has 6 equally likely outcomes :

$S = \{$ Card 1 selected, face R_1 shown $\leftarrow$ a

" 1 " , " R_2 " $\leftarrow$ b

" 2 " , " B_1 " $\leftarrow$ c

" 2 " , " B_2 " $\leftarrow$ d

" 3 " , " R " $\leftarrow$ e

" 3 " , " B " $\leftarrow$ f $\}$

Given information that red face shown, sample space collapses to 3 outcomes : a, b, e.

Of these 3, 2 satisfy the condition that hidden face is red : a, b.

So probability is $\dfrac{2}{3}$

3.25) $F = \{\text{female}\}$
$C = \{\text{CS major}\}$

a) $$P(F|C) = \frac{P(FC)}{P(C)} = \frac{.02}{.05} = .4$$

b) $$P(C|F) = \frac{P(CF)}{P(F)} = \frac{.02}{.52} = .038\ldots$$

3.27) $D_i = \{i^{th} \text{ radio defective}\}$

$$P(D_2|D_1) = \frac{P(D_1D_2)}{P(D_1)}$$

$$= \frac{P(D_1D_2|A)P(A) + P(D_1D_2|B)P(B)}{P(D_1|A)P(A) + P(D_1|B)P(B)}$$

$$= \frac{(.05)^2 \times .5 + (.01)^2 \times .5}{(.05) \times .5 + (.01) \times .5} \approx .0433$$

3.29) $A = \{\text{Alive}\}$, $W = \{\text{watered}\}$

a) $$P(A) = P(A|W)P(W) + P(A|W^c)P(W^c)$$
$$= .85 \times .9 + .2 \times .1 = .785$$

b) $$P(W^c|A^c) = \frac{P(W^cA^c)}{P(A^c)} = \frac{P(A^c|W^c)P(W^c)}{P(A^c)}$$

$$= \frac{.8 \times .1}{1 - .785} \approx \cancel{.372}\ .372$$

3.35) $P(A_c) = P(A_c|G)P(G) + P(A_c|A_v)P(A_v) + P(A_c|B)P(B)$

$= .05 \times .2 + .15 \times .5 + .3 \times .3$

$= .175$

($A_c = \{\text{has accident}\}$, $G = \{\text{Good risk}\}$, etc.)

$$P(G|A_c^c) = \frac{P(A_c^c|G)P(G)}{P(A_c^c)} \quad \text{(Bayes)}$$

$$= \frac{.95 \times .2}{1 - .175} = .23\ldots$$

$$P(A_v|A_c^c) = \frac{.85 \times .5}{.825} = .51\ldots$$

3.37) For a particular configuration, say

Relays 1, 3 closed

Relays 2, 4, 5 open,

appropriate probability to assign is

$$\underset{\substack{\uparrow \\ \text{closed}}}{p_1}\ \underset{\substack{\uparrow \\ \text{open}}}{(1-p_2)}\ p_3 (1-p_4)(1-p_5)$$

(this captures independence and gives right probabilities for individual relays)

For each of the three circuits, need to

i) list all possible configurations that lead to current flowing

ii) assign individual probabilities to each configuration, as described

iii) add up all these probabilities.

Here I'll just list the configurations that lead to current flowing (✓ indicates closed relay, ✗ indicates open)

a)

1	2	3	4	5
✓	✓	✓	✗	✗
✓	✓	✓	✓	✗
✓	✓	✓	✗	✓
✓	✓	✓	✓	✓
✗	✗	✗	✓	✓
✓	✗	✗	✓	✓
✗	✓	✗	✓	✓
✗	✗	✓	✓	✓
✓	✓	✗	✓	✓
✓	✗	✓	✓	✓
✗	✓	✓	✓	✓

b)

1	2	3	4	5
✓	✓	✗	✗	✓
✓	✓	✗	✓	✓
✓	✓	✓	✗	✓
✓	✓	✓	✓	✓
✗	✓	✓	✓	✓
✓	✗	✓	✓	✓
✗	✗	✓	✓	✓

c)

1	2	3	4	5
✓	✓	✗	✗	✗
✓	✓	✗	✗	✓
✓	✓	✗	✓	✗
✓	✓	✗	✓	✓
✗	✓	✗	✓	✓
✓	✗	✗	✓	✓
✗	✗	✗	✓	✓

1	2	3	4	5
✓	✗	✓	✓	✗
✓	✗	✓	✗	✓
✓	✗	✓	✓	✓
✗	✓	✓	✓	✗
✗	✓	✓	✗	✓
✗	✓	✓	✓	✓
✓	✓	✓	✓	✗
✓	✓	✓	✗	✓
✓	✓	✓	✓	✓

3.39) a) $P(\text{first three are } H) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$P(\text{" " " } T) = \frac{1}{8}$

$P(\text{first three the same}) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

b) $F = \{\text{first three the same}\}$

$L = \{\text{last " " "}\}$

Want $P(F \cup L)$.

$$P(F \cup L) = P(F) + P(L) - P(FL)$$

$$= \frac{1}{4} + \frac{1}{4} - \frac{1}{16} = \frac{7}{16}$$

$[\,P(FL) = P(HHHHH \text{ or } TTTTT)\,]$

c) Possible good outcomes:

HHHTT
HHTTT
HHTHT
HHTTH
HTHTT
THHTT

All equally likely, so total probability is $\frac{6}{32}$.

3.47) a) $P(A \cup B) = P(A) + P(B) - P(AB)$
$= P(A) + P(B) - 0$
$= .5$

b) $P(A \cup B) = P(A) + P(B) - P(AB)$
$= P(A) + P(B) - P(A)P(B)$
$= .44$

c) $P(ABC) = P(A)P(B)P(C)$
$= .024$

d) $P(ABC) = 0$

4.1) 10! possible orderings.

orderings in which first women is in place 1:

$$\underset{\substack{\uparrow \\ \text{pick woman to be in place 1}}}{5} \times 9! \longleftarrow \text{arrange everyone else arbitrarily}$$

So $P(X=1) = \dfrac{5 \times 9!}{10!} = .5$

orderings in which first woman is in place 2:

$$\underset{\substack{\uparrow \\ \text{man first}}}{5} \times \underset{\substack{\uparrow \\ \text{woman second}}}{5} \times 8!$$

So $P(X=2) = \dfrac{5 \times 5 \times 8!}{10!} = .2\dot{7}$

Similarly,

$$P(X=3) = \frac{5 \times 4 \times 5 \times 7!}{10!} = .1389$$

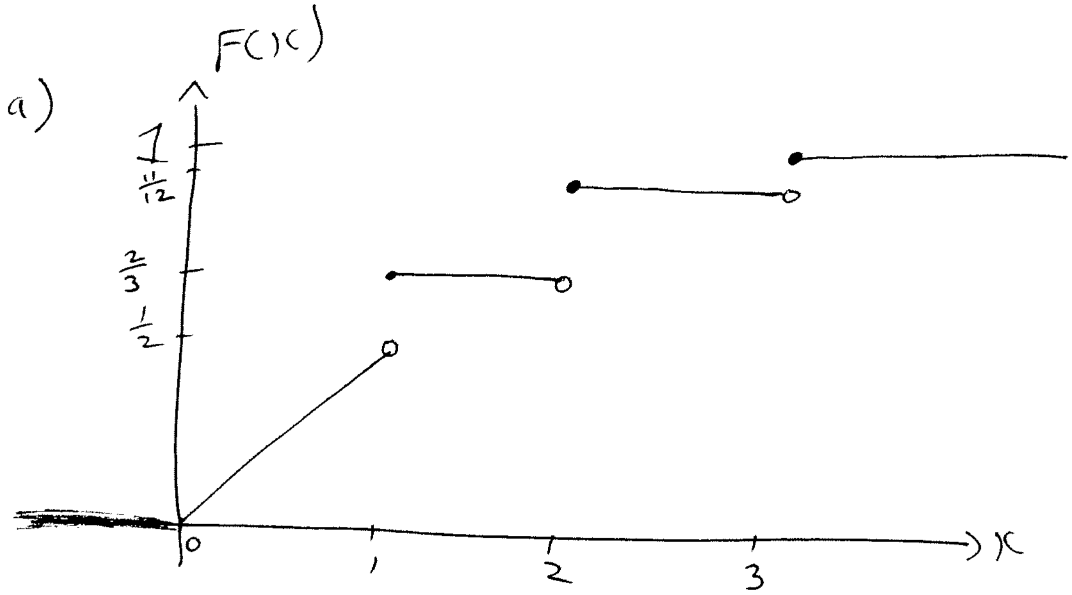
$$P(X=4) = \frac{5 \times 4 \times 3 \times 5 \times 6!}{10!} = .0595$$

$$P(X=5) = \frac{5 \times 4 \times 3 \times 2 \times 5 \times 5!}{10!} = .0198$$

$$P(X=6) = \frac{5! \times 5!}{10!} = .004$$

$$P(X=7) = P(X=8) = P(X=9) = P(X=10) = 0$$

4.4) a)



b) $P(X > \frac{1}{2}) = 1 - \underbrace{P(X \leq \frac{1}{2})}_{= F(\frac{1}{2})}$

$= 1 - \frac{1}{4} = \frac{3}{4}$

c) $P(2 < X \leq 4) = P(X \leq 4) - P(X \leq 2)$

$= F(4) - F(2)$

$= 1 - \frac{11}{12} = \frac{1}{12}$

d) $P(X < 3) = F(\text{just before } 3) = \frac{11}{12}$

e) $P(X = 1) = \text{Amount } F \text{ jumps at } 1$

$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$