

Math 30440 — Probability and Statistics

Spring 2010 second mid-term exam practice problems 2

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1. The time until the next large earthquake (magnitude greater than 4.8) occurs in the United States is an exponentially distributed random variable. Suppose that the probability that the next large earthquake will occur sometime in the next 200 days is exactly .5. What is the expected length of time until the next large earthquake?
2. Each fall my wife participates in a football pool at work with seven other people. Each week during the Colts' 16 week regular season, each person pays in \$2 (for a total of \$16), and the \$16 dollars goes to one person in a competition based on the final score of that week's Colts game. Assume that competitions from week to week are independent, and that each week each person has a $1/8$ chance of winning.
 - (a) Let X be the number of times my wife wins during the 16 week season. What is the expectation and variance of X ?
 - (b) What is $P(X \geq 2)$? (Since she puts in a total of \$32, this is the probability that she at least breaks even in the football pool.)
 - (c) Let Y be the number of people in the pool who at least break even over the whole 16 weeks. Compute $E(Y)$.
3. A grain dispenser can be set to fill containers of different sizes. If it is set to dispense μ kilos, then the amount it dispenses is a normal random variable with mean μ and standard deviation 20.
 - (a) Suppose μ is set to 970, and the dispenser is used to fill containers that hold 1000 kilos. What is the probability that a container will be filled to overflowing?
 - (b) What should μ be set to, to make sure that the probability of a 1000 kilo container being filled to overflowing is only 2%?
4. A college gives each of the entering first years a laptop, promising to buy any working laptops back in four years. Suppose that the lifetime of such laptops

is exponentially distributed with a mean lifetime of 3 years. After 2 years the college discovers that 1000 of the initial laptops are still functioning. Help the administration compute:

- (a) The probability that a particular laptop that is still working after after 2 years, will also still be working after 4 years.
 - (b) The approximate probability that the college will have to buy back more than 500 laptops (assuming the lifetimes of the laptops are independent).
5. Car insurance claims are normally distributed with a mean of \$2,100 and a standard deviation of 100. A claim is classified as high-risk if it exceeds \$2,250.
- (a) What is the percentage of high-risk claims?
 - (b) A claim is said to be of highest risk if the amount is in the top 2% of all claims. For what amounts would a claim be classified as being of the highest risk?
 - (c) If a person files two independent claims, what is the probability the total sum will exceed \$4,500?
6. The number of years a brand new TV set is suppose to function is modeled by an exponential random variable with mean 10 years.
- (a) What is the probability that it works for at least 5 years?
 - (b) What is the probability that if it worked for 10 years it will work for at least an additional 5 years? Explain your reasoning.
 - (c) Answer the last two parts again if the lifetime of the TV set is modeled on a uniform random variable with the same mean as before.
7. A certain biased coin comes up heads 70% of the time. It is flipped 1000 times. What is the approximative probability that the number of tails will be fewer than 320?