MATH 30440, SEC 01, SPRING 2010

EXAM 2 PRACTICE PROBLEMS 2 – SOLUTIONS

1) X = time 'til earthquake
= exponential (λ) $\quad$ [λ = average # per day]

To find λ:

$$P(X \leq 200) = .5$$

So $$\int_0^{200} \lambda e^{-\lambda x}\,dx = .5$$

$$\left[-e^{-\lambda x}\right]_0^{200} = .5$$

$$1 - e^{-200\lambda} = .5$$

$$\lambda = \frac{\ln 2}{200}$$

So $$E(X) = \frac{200}{\ln 2}$$

2) a) $X = \text{Binomial}, \; n = 16, \; p = \frac{1}{8}$

$E(X) = 2, \quad Var(X) = \frac{14}{8}$

b) $P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$= 1 - \binom{16}{0}\left(\frac{7}{8}\right)^{16} - \binom{16}{1}\left(\frac{1}{8}\right)\left(\frac{7}{18}\right)^{15}$

$= .6121$

c) $Y_i = \begin{cases} 1 & \text{if person } i \text{ breaks even} \\ 0 & \text{owise} \end{cases}$

By part b), $E(Y_i) = .6121$

$E(Y) = \sum E(Y_i) = 8 \times .6121$

$= 4.8968$

3) X = amount dispersed

$= \text{Normal}(\mu, 400)$

a) $\mu = 970$

$$P(X > 1000) = P\left(Z > \frac{1000 - 970}{20}\right)$$

$$= P(Z > 1.5) = .0668$$

b) Want $P\left(Z > \frac{1000-\mu}{20}\right) = .02$

Since $P(Z > 2.05) = .02$,

want $\frac{1000 - \mu}{20} = 2.05$, $\mu = 959$

4) a) X = lifetime of laptop

= exponential with $\lambda = \frac{1}{3}$

$$P(X > 4 \mid X > 2) = P(X > 2) \quad (\text{memory-lessness})$$

$$= \int_2^\infty \frac{1}{3} e^{-\frac{x}{3}} dx$$

$$= \left[-e^{-\frac{x}{3}}\right]_2^\infty$$

$$= e^{-\frac{2}{3}}$$

b) Y = # bought back

= Binomial, $n = 1000$, $p = e^{-\frac{2}{3}}$

surviving after 2 years

$$E(Y) = 1000 e^{-\frac{2}{3}}.$$

5) X = amount of claim

a) $P(X > 2250) = P(Z > 1.5)$

$= .0668$

b) $P(Z > 2.05) = .02$

So "highest risk" is 2.05 std devs above mean, or greater

So cutoff is 2305

c) X_1 = first claim = Normal $(2100, (100)^2)$

X_2 = second = Normal $(2100, (100)^2)$

$X_1 + X_2$ = Normal $(4200, 20000)$

$P(X_1 + X_2 > 4500)$

$= P\left(Z > \frac{300}{\sqrt{20000}}\right) = P(Z > 2.12)$

$= .0166$

6) a) $\lambda = \frac{1}{10}$ (Since Mean $= 10$)

$$P(X>5) = \int_5^\infty \frac{1}{10} e^{-\frac{x}{10}}\,dx$$

$$= \left[-e^{-\frac{x}{10}}\right]_5^\infty$$

$$= e^{-\frac{1}{2}}$$

b) By memorylessness, still $e^{-\frac{1}{2}}$

c) Uniform $(0, 20)$ has Mean 10

$$P(X>5) = \frac{3}{4}$$

$$P(X>15 \mid X>5) = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

7) $X = \#$ fails $=$ binomial, $n = 1000$
$p = .3$

Mean 300, variance 210

$X \approx \text{Normal}(300, 210)$

$$P(X < 320) = P\left(Z < \frac{20}{\sqrt{210}}\right) = P(Z < 1.38)$$

$$= .9162$$