Math 30440 — Probability and Statistics

Spring 2010 second mid-term exam practice problems

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1. Let X and Y be continuous independent random variables, with X uniformly distributed on $[0,1]$ and Y exponentially distributed with $\lambda = 1$.

   Find $P(X + Y \geq 1)$.

2. One-tenth of 1% of a certain type of RAM chip are defective. A student needs 50 chips for a certain board.

   (a) What is the probability that if she buys 53 chips, she'll have enough working chips for the board?

   (b) What is the smallest number of chips that she should buy in order for there to be at least a 99% chance of having at least 50 working chips?

3. A soft drink machine can be regulated so that it discharges an average of μ ounces per cup. If the ounces of fill are described by a normal random variable with standard deviation $\sigma = 0.3$ ounce, find a value of μ such that 8-ounce cups will overflow only 1% of the time.

4. The distribution of resistance for resistors of a certain type is known to be normal. 9.85 % of all resistors have a resistance exceeding 10.257 Ohms, and 5.05 % have resistance smaller than 9.671 Ohms. What are the mean value and variation of the resistance distribution?

5. Let X be a Poisson random variable with parameter $\lambda = 2$. Find the expected value of $Y = 2^X$.

6. If X is a random variable uniformly distributed over the interval $(0, 2)$, find $E(X^3 - X)$

7. An average of 10 cars per hour stop at a gas station. Arrivals of different cars are independent of each other.

   (a) What is the probability that at most one car will stop at the gas station during the next 30 minutes?

(b) What is the probability that the first car of the hour will arrive no more than 10 minutes into the hour?

8. One microgram of radium contains 10^{16} atoms. The probability that a single atom will disintegrate during a one-millisecond time is $p = 10^{-15}$. Use the Poisson random variable to approximate the probability that more than two atoms will disintegrate in one millisecond.

9. Kyle and Gordan have a contest to determine who has the lost the most weight over the course of a month. Using their own scales they each weigh themselves at the start and end of the week to determine how much weight they have lost, and then compare these values to determine a winner. By their measurements, Gordon has gone from 210 lbs to 202 lbs and Kyle has gone from 216 lbs to 210 lbs. Suppose that Gordon's scale gives an error that is distributed normally with mean .5 and standard deviation .8 and Kyle's scale gives an error that is distributed normally with mean $-.2$ and standard deviation .6 What is the probability that Gordon has actually lost more weight than Kyle?

10. If the final position of a billiard ball hit hard on a 100" by 50" pool table is uniform, compute the probability that the ball comes to rest within 20" of the center of the table.

11. Suppose that there are an average of 50 house fires per month in Michiana. If we assume that houses catch fire independently of each other, write an expression for the probability that there will be exactly 15 house fires next month.

12. Suppose that Charlie's dog Fido runs off into the forest and painful past experience has taught Charlie that the time until his dog is sprayed by a skunk is exponentially distributed with a mean of 1 hour. If the time it takes Charlie to race back home and return with Fido's favorite toy ranges between 15 and 30 minutes and is uniformly distributed over that interval give an expression for the probability that Charlie can lure Fido back with his favorite toy before he gets sprayed by a skunk.

13. The life of fan belt in a car engine (measured in miles) is distributed exponentially with parameter $\lambda = \frac{1}{30000}$. If a fan belt is observed to be in working order $25,000$ miles after installation, how many more miles is it expected to work for?

14. The average income of Notre Dame alumni (measured in thousands of dollars) is 80, with variance 120. Thirty Domers find themselves sitting randomly together on game day, and pass the time during a media time out by computing their average annual income.

    How likely is it that they get an answer greater than 85?