7.23) $\overline{X} \pm t_{.025,\infty} \frac{s}{\sqrt{n}} = (1124.5\ldots, 1315.4\ldots)$

7.24) We want a 90% one-sided confidence interval of the form $(-\infty, r)$

$$r = \overline{X} + t_{.10,\infty} \frac{s}{\sqrt{n}} = 1220 + 1.284 \frac{840}{\sqrt{300}}$$

$$= 1282.27\ldots$$

7.28) $\overline{X} = .55$

$s^2 = .000866\ldots$

95% confidence interval is $\overline{X} \pm t_{.025,9} \frac{s}{\sqrt{10}}$

or $(.529, .571)$

7.34) We want a confidence interval of the form $(-\infty, \text{SOMETHING})$

$$\text{SOMETHING} = \overline{X} + t_{.10,29} \frac{s}{\sqrt{n}}$$

$$= 2.5 + 1.311 \frac{2.12}{\sqrt{30}}$$

$$= 3.007\ldots$$

7.37) $\bar{X} = 6.7236$ $\qquad n = 36$

$s^2 = .00312$

A 95% 2-sided confidence interval is

$$\left(\frac{35s^2}{\chi^2_{.025,35}} , \frac{35s^2}{\chi^2_{.975,35}} \right)$$

The table does not give critical values for χ^2 with $n = 35$; we can deduce from the table though that

$\chi^2_{.025,35} \approx 53$ (by around $n = 30$, the values in the .025 column are increasing by about 1.2 each time n increases by 1)

$\chi^2_{.975,35} \approx 20$ ($\chi^2_{.975,30} \approx 16.75$, and .975 values are increasing by around .75 each time n increases by 1)

So confidence interval is roughly

$\cancel{(20.6, 54.7)}$

$(.00206, .00529)$

7.39) $\overline{X} = 3.178375$

$$s^2 = \frac{\sum_{i=1}^{8} (x_i - 3.178375)^2}{7}$$

$$\approx .000064$$

So $\sigma \approx .008$, and a 90% confidence interval estimate for σ is

$$\left(\sqrt{\frac{7s^2}{\chi^2_{.05,7}}}, \sqrt{\frac{7s^2}{\chi^2_{.95,7}}} \right) = (.0056, .0144)$$

7.41) $\overline{X}_1 = 3358.1$ $n = 10$

$S_1^2 = 124419.2$

$\overline{X}_2 = 3130.4$ $m = 10$

$S_2^2 = 17728.71$

$$S_p^2 = \frac{9S_1^2 + 9S_2^2}{18} = 71073.96$$

a) 95% confidence interval for $\mu_1 - \mu_2$:

$$\overline{X}_1 - \overline{X}_2 \pm t_{.025,18}\sqrt{\frac{S_p^2}{10} + \frac{S_p^2}{10}}$$

$= (-22.84, 478.24)$

b) Upper confidence interval:

$$\left(\overline{X}_1 - \overline{X}_2 - t_{.05,18}\sqrt{\frac{S_p^2}{10} + \frac{S_p^2}{10}}, \infty\right)$$

$= (20.91, \infty)$

c) Lower confidence interval

$$\left(-\infty, \overline{X}_1 - \overline{X}_2 + t_{.05,18}\sqrt{\frac{S_p^2}{10} + \frac{S_p^2}{10}}\right)$$

$= (-\infty, 434.49)$

7.44) $\bar{X}_1 = 532.1$ $\qquad n = 10$

$S_1^2 = 2933$

$\bar{X}_2 = 548.6$

$S_2^2 = 1192$ $\qquad m = 10$

$$S_p^2 = \frac{9S_1^2 + 9S_2^2}{18} = 2062$$

99% confidence interval of $\mu_1 - \mu_2$:

$$\bar{X}_1 - \bar{X}_2 \pm t_{.005,18}\sqrt{\frac{S_p^2}{n} + \frac{S_p^2}{m}}$$

$= (-74.97, 41.97)$

7.49) $100(1-\alpha)\%$ confidence interval: (approximate)

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

a) $\hat{p} = .5106$, $n = 10\,000$, $\alpha = .10$, $z_{\alpha/2} = 1.64$

$(.5024, .5188)$

b) $\alpha = .01$, $z_{\alpha/2} = 2.58$

$(.4977, .5235)$

7.55) $\hat{p} = .17$

95% two-sided confidence interval:

$$\hat{p} \pm z_{.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = (.096, .244)$$

99% upper confidence interval:

$$\left(\hat{p} - z_{.01}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, 1\right) = (.0825, 1)$$

Assumptions: 1) Sample size is large enough to apply CLT

2) Samples were chosen independently

8.3) Test statistic : $\dfrac{\bar{X} - 50}{\frac{20}{\sqrt{64}}}$

a) $\bar{X} = 52.5$, $TS = 1$, p-value is

$P(Z > 1 \text{ or } Z < -1) = .3174$

b) $\bar{X} = 55$, $TS = 2$, p-value is

$P(Z > 2 \text{ or } Z < -2) = .0455$

c) $\bar{X} = 57.5$, $TS = 3$, p-value is

$P(Z > 3 \text{ or } Z < -3) = .0027$

8.4) $\bar{X} = 8.179$

If null ($H_0 : \mu = 8.20$) is true, then

$\dfrac{\bar{X} - 8.20}{.02/\sqrt{10}} = Z$; i.e, 3.32 is a reading from a standard normal.

p-value : $P(Z > 3.32 \text{ or } Z < -3.32) = .001$

a) Reject null

b) Reject null.

8.4) a) Two sided test $H_0 : \mu = 8.2$

$H_1 : \mu \neq 8.2$

b) Suppose we have sample size n.

We will accept H_0 if

$$-1.96 \leq \frac{\bar{X} - 8.2}{\frac{.02}{\sqrt{n}}} \leq 1.96$$

first line of problem tells us that this is a 5% significance test

ie, if $\boxed{\cancel{\frac{.0392}{\sqrt{n}}}}$ $8.2 - \frac{.0392}{\sqrt{n}} \leq \bar{X} \leq 8.2 + \frac{.0392}{\sqrt{n}}$

We want to arrange that if the true mean is actually 8.23, we reject the null with probability $.95$, ie we accept null with probability $.05$.

But if true mean is 8.23, then

$$\bar{X} = N\left(8.23, \frac{(.02)^2}{n}\right)$$

So we want to choose n so that

$$P\left(8.2 - \frac{.0392}{\sqrt{n}} \leq N\left(8.23, \frac{(.02)^2}{n}\right) \leq 8.2 + \frac{.0392}{\sqrt{n}}\right)$$

$$= .05$$

ie $P\left(\dfrac{-.03 - \frac{.0392}{\sqrt{n}}}{\frac{.02}{\sqrt{n}}} \le Z \le \dfrac{-.03 + \frac{.0392}{\sqrt{n}}}{\frac{.02}{\sqrt{n}}} \right) = .05$

ie, $P(-1.5\sqrt{n} - 1.96 \le Z \le -1.5\sqrt{n} + 1.96) \le .05$

This is roughly

$$P(Z \le -1.5\sqrt{n} + 1.96) \le .05$$

i.e. $\quad -1.5\sqrt{n} + 1.96 = -1.645$

$$n = \frac{(1.645 + 1.96)^2}{(1.5)^2} = 5.776\ldots$$

So should take $n = 6$;

Similarly, if true mean is 8.17, should take $n = 6$ to detect this with probability at least .95.

We could also have applied formula (8.3.7) on page 299: $\alpha = .05$ (Significance)

$\beta = .05$ (Desired max probability of type II error)

$\mu_1 - \mu_0 = .03$ ($\mu_0 = 8.2$ is null hypothesised mean; $\mu_1 = 8.23$ or 8.17; true mean)

$\sigma = .02$

$$n \approx \frac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{(\mu_1 - \mu_0)^2} = \frac{(1.96 + 1.645)^2 (.02)^2}{(.03)^2}$$

$$= 6.$$

c) With $H_0 : \mu = 8.20$ $\quad n = 6,$ $\quad \sigma = .02$

$H_1 : \mu \neq 8.2$ $\quad \bar{X} = 8.31$

TS is $\dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = 13.47$; Reject at any significance

d) $< 5\%$, since we have chosen a sample size large enough that we reject null 95% of time even when true mean is 8.23; 8.32 is much larger.

8.11) $H_0: \mu \leq 100$ $\quad n = 20$

$H_1: \mu > 100$ $\quad \bar{X} = 105$

Test statistic: $\dfrac{105 - 100}{\frac{\sigma}{\sqrt{20}}} = \dfrac{22.36}{\sigma}$

~~We will reject null at~~

p-value of test is $P\left(Z \geq \dfrac{22.36}{\sigma}\right)$

a) $\sigma = 5$, $P(Z \geq 4.47) = 0$

b) $\sigma = 10$, $P(Z \geq 2.236) = .0127$

c) $\sigma = 15$, $P(Z \geq 1.491) = .068$

8.12) μ = mean # cavities with new toothpaste

$H_0 : \mu < 3$

$H_1 : \mu = 3$ (or $\mu \geq 3$) $\quad \sigma = 1$

$n = 2500$, $\bar{X} = 2.95$

Test statistic $\dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \dfrac{2.95 - 3}{\frac{1}{\sqrt{2500}}} = -2.5$

If null is true, this is reading from Z, a standard normal

p-value: $P(Z \leq -2.5)$ (one-sided test)

$= .0062$

a) Accept alternative (H_1) at 5%

b) Yes, although the drop in average # of cavities from 3 to 2.95 is not that spectacular.