7.1) Joint distribution of n readings:

$$f(\theta) = \prod_{i=1}^{n} e^{-(x_i-\theta)} \quad \left(\begin{array}{l} \text{if } \underline{\text{all}} \; x_i \geq \theta; \\ 0 \text{ otherwise} \end{array} \right)$$

$$= e^{n\theta - \Sigma x_i}$$

Make this large by making θ large;

but θ has to be $\leq$ <u>all</u> x_i.

So get maximum at $\hat{\theta}$ = minimum of x_i

7.3) Joint density of n readings:

$$f(\sigma) = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^n e^{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}}$$

$$\log f = n \log \frac{1}{\sqrt{2\pi}} - n \log \sigma - \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}$$

$$(\log f)' = \frac{-n}{\sigma} + \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{\sigma^3}$$

$$= 0 \text{ when } \longrightarrow \sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$$

This is MLE

$$E\left(\frac{\sum_{i=1}^{n}(X_i-\mu)^2}{n}\right) = \frac{\sum_{i=1}^{n} E(X_i-\mu)^2}{n}$$

$$= \frac{n\,\mathrm{Var}(X_i)}{n} = \sigma^2$$

7.4) Height of tower $= X \tan\Theta$

MLE for X is $\overline{X} = 150.456$

MLE for Θ is $\overline{\Theta} = 40.27$

Best guess for height:

$150.456 \tan 40.27 \approx 127.5$

7.8) $\overline{X} = 3.1502$, so a $100(1-\alpha)\%$ confidence interval for true weight is

$$\left(\overline{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \overline{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

$$= (3.1502 - z_{\alpha/2} \cdot .04472,\ 3.1502 + z_{\alpha/2} \cdot .04472)$$

a) $\alpha = .05$, $z_{\alpha/2} = 1.96$, get

$(3.0625, 3.2379)$

b) $\alpha = .01$, $z_{\alpha/2} = 2.58$, get

$(3.0348, 3.2656)$

7.9) $\bar{X} = 11.48$

a) $\left(11.48 \pm 1.96 \frac{.08}{\sqrt{10}}\right) = (11.43, 11.53)$

b) $\left(-\infty, 11.48 + 1.645 \frac{.08}{\sqrt{10}}\right) = (-\infty, 11.5216)$

c) $\left(11.48 - 1.645 \frac{.08}{\sqrt{10}}, +\infty\right) = (11.4384, \infty)$

7.13) $\left(1.2 \pm 2.58 \frac{.2}{\sqrt{20}}\right) = (1.0848, 1.3152)$

7.17) $\bar{X} = 333.995\ldots$

$S^2 = 48.417\ldots$

95% confidence interval:

$\bar{X} \pm t_{23,.025} \frac{S}{\sqrt{24}} = (331.0572, 336.9345)$

99% confidence interval

$\bar{X} \pm t_{23,.005} \frac{S}{\sqrt{24}} = (330.0082, 337.9836)$

7.23) $\bar{X} \pm t_{.025,\infty} \frac{S}{\sqrt{n}} = (1124.5\ldots, 1315.4\ldots)$

7.24) We want a 90% one-sided confidence interval of the form $(-\infty, r)$

$$r = \bar{X} + t_{.10,\infty} \frac{S}{\sqrt{n}} = 1220 + 1.284 \frac{840}{\sqrt{300}}$$

$$= 1282.27\ldots$$

7.28) $\bar{X} = .55$

$S^2 = .000866\ldots$

95% confidence interval is $\bar{X} \pm t_{.025,9} \frac{S}{\sqrt{10}}$

or $(.529, .571)$

7.34) We want a confidence interval of the form $(-\infty, \text{SOMETHING})$

$$\text{SOMETHING} = \bar{X} + t_{.10,29} \frac{S}{\sqrt{n}}$$

$$= 2.5 + 1.311 \frac{2.12}{\sqrt{30}}$$

$$= 3.007\ldots$$