

Math 30440, Homework 6 Solutions
(all problems from Ross, Chapter 5)

$$23) b) P(4 < X < 16) = P\left(\frac{4-10}{6} < \frac{X-10}{6} < \frac{16-10}{6}\right)$$

Using: $X = N(10, 36)$ $= P(-1 < Z < 1)$

$$X = 6Z + 10$$

$$= \Phi(1) - \Phi(-1)$$

$$= \Phi(1) - [1 - \Phi(1)]$$

$$= .6827$$

$$c) P(X < 8) = \Phi\left(-\frac{1}{3}\right)$$

$$= 1 - \Phi\left(\frac{1}{3}\right)$$

$$= .3695$$

$$e) P(X > 16) = 1 - \Phi(1)$$

$$= .1587$$

(All numerical answers taken from standard normal table at back of book)

$$\begin{aligned} 24) \quad a) & P(\text{A particular student scores below } 600) \\ &= P(Z < 1) \\ &= .84\dots \end{aligned}$$

$$\begin{aligned} \text{So } P(\text{all 5 do}) &= (.84\dots)^5 \\ &= .4215 \end{aligned}$$

$$\begin{aligned} b) & P(\text{particular student scores above } 640) \\ &= P(Z > 1.4) \end{aligned}$$

$$\begin{aligned} \text{So } P(\text{particular student scores below } 640) \\ &= \Phi(1.4) \end{aligned}$$

$$\begin{aligned} \text{and } P(\text{exactly 3 above } 640) \\ &= P(\text{exactly 2 below } 640) \\ &= P(\text{Binomial}(5, \Phi(1.4)) = 2) \\ &= \binom{5}{2} \Phi(1.4)^2 (1 - \Phi(1.4))^3 \end{aligned}$$

$$= .0045$$

27) $X = \text{lifetime} = \text{Normal}(2000, (85)^2)$

Want L so that

$$P(X \geq L) = .95$$

$$\begin{aligned} \text{But } P(X \geq L) &= P\left(\frac{X - 2000}{85} \geq \frac{L - 2000}{85}\right) \\ &= P\left(Z \geq \frac{L - 2000}{85}\right) \end{aligned}$$

From table, $P(Z \geq -1.64) = .95$

So want L s.t. $\frac{L - 2000}{85} = -1.64,$

$$L = 1860.6$$

$$31) X = N(4.4 \times 10^6, (3 \times 10^5)^2)$$

$$P(X \geq 4 \times 10^6) = P\left(\frac{X - 4.4 \times 10^6}{3 \times 10^5} \geq \frac{4 \times 10^6 - 4.4 \times 10^6}{3 \times 10^5}\right)$$

$$= P\left(Z \geq -\frac{4}{3}\right)$$

$$= .9091 \dots$$

So w probability $> 90\%$, a chip meets the specification ... manufacturer should contract

$$34) X = N(40.14, (8.7)^2)$$

$$a) P(X > 42) = \boxed{.415}$$

b) $X_1 =$ rainfall this year, $X_2 =$ next year

$$X_1 + X_2 = N(80.28, 2(8.7)^2)$$

$$P(X_1 + X_2 > 84) = \boxed{.381}$$

$$c) P(X_1 + X_2 + X_3 > 126) =$$

$$P(N(120.42, 3(8.7)^2) > 126)$$

$$= \boxed{.356}$$

d) Assuming rainfall amounts in different years are independent.

$$37) a) P(X > 2) = \int_2^{\infty} e^{-x} dx$$

Density of X:

$$= [-e^{-x}]_2^{\infty}$$

$$f(x) = e^{-x}, x \geq 0$$

$$= e^{-2} = \boxed{.135\dots}$$

$$b) P(X > 3 | X > 2) = P(X > 1) \text{ (memorylessness)}$$

$$= e^{-1} = \boxed{.367\dots}$$

39) X = lifetime = exponential ($\frac{1}{20}$)

P(Jones gets 20,000 miles | Car already has 10,000)

$$= P(X > 30 \text{ } | X > 10)$$

$$= P(X > 20) \text{ (Memorylessness)}$$

$$= e^{-1} = \boxed{.367\dots}$$

IF X is uniform (0, 40),

$$P(X > 30 | X > 10) = \frac{P(X > 30)}{P(X > 10)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \boxed{\frac{1}{3}}$$