MATH 30440, SPRING 2009 Homework 1 Solutions

1) $S = \{RR, RB, RG, BR, BG, BB, GR, GB, GG\}$

(where "RR" means "first marble is red, second marble is red)

For the second experiment,

$S = \{RB, RG, BR, BG, GR, GB\}$

4) EF = First is 1, and sum is odd

$E \cup F$ = Either first is 1 or sum is odd (or perhaps both)

FG = First is 1 and sum is 5

EF^c = Sum is odd and not first is 1

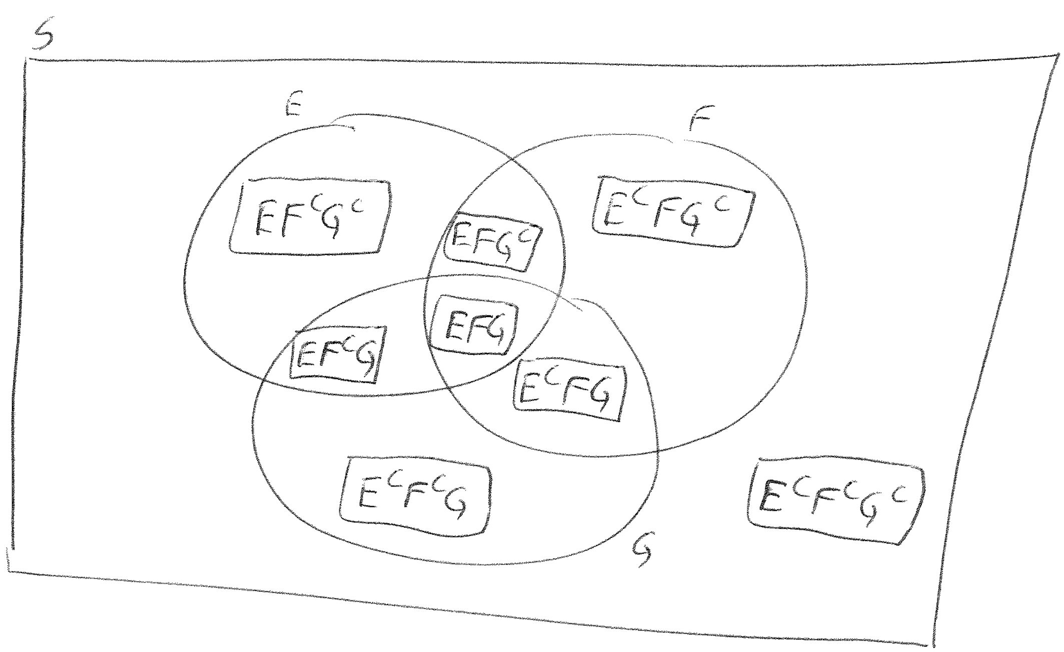
EFG = Sum is odd and first is one and sum is 5

5) a) $2^4 = 16$

b) $(1,1,0,0), (1,1,1,0), (1,1,0,1), (1,1,1,1)$
$(0,1,1,1), (1,0,1,1), (0,0,1,1)$.

c) $2^2 = 4$ [2 choices for each of components 2 and 4]

6)



a) EF^cG^c

b) EF^cG

c) Union of all of the boxed expressions above, <u>except</u> $E^cF^cG^c$

d) $EFG^c \cup EF^cG \cup E^cFG \cup EFG$

e) EFG

f) $E^cF^cG^c$

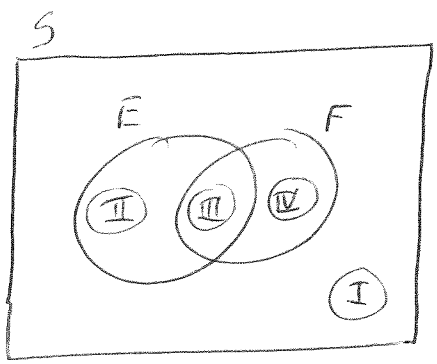
g) $E^cF^cG^c \cup EF^cG^c \cup E^cFG^c \cup E^cF^cG$

h) Union of everything <u>except</u> EFG

i) $EFG^c \cup EF^cG \cup E^cFG$

j) Union of everything

8)



a) $EF = \text{III}$, $E = \text{II} \cup \text{III}$, so $EF \subseteq F$

$E \cup F = \text{II} \cup \text{III} \cup \text{IV}$, so $E \subseteq E \cup F$

b) If $E \subseteq F$ then there is nothing in $\cancel{\text{IV}}$ II

$F^c = \text{I} \cup \text{II} = \text{I}$ (if there is nothing in II)

$E^c = \text{I} \cup \text{IV}$

So $F^c \subseteq E^c$

c) $EF = FE = \text{III}$

$E \cup F = F \cup E$ = everything in II, III and IV

e) $FE = \text{III}$, $FE^c = \text{IV}$, so

$FE \cup FE^c = \text{III} \cup \text{IV} = F$

f) $E \cup F = \text{II} \cup \text{III} \cup \text{IV}$

$E \cup E^cF = \text{II} \cup \text{III} \cup \text{IV}$

} so they are equal

g) $(E \cup F)^c = (\text{II} \cup \text{III} \cup \text{IV})^c = \text{I}$

$E^cF^c = \text{I}$

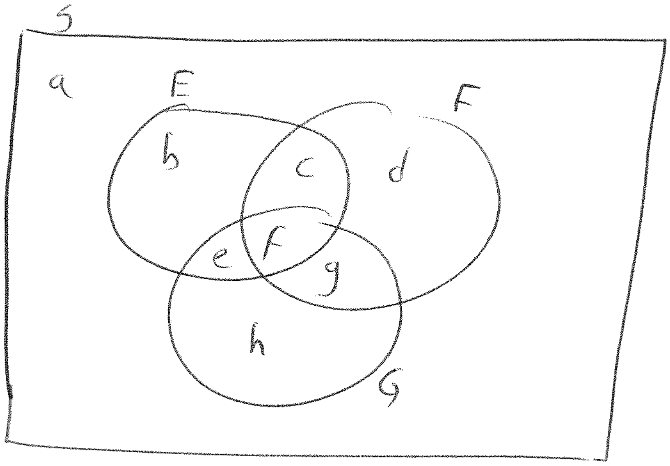
} so they are equal

$(EF)^c = \text{III}^c = \text{I} \cup \text{II} \cup \text{IV}$

$E^c \cup F^c = \text{I} \cup \text{II} \cup \text{IV}$

} so they are equal

Picture for d):



$(EF)G = (c \cup f)(e \cup f \cup g \cup h) = f$

$E(FG) = (b \cup c \cup e \cup f)(f \cup g) = f$

$(E \cup F) \cup G = (b \cup c \cup d \cup e \cup f \cup g) \cup (e \cup f \cup g \cup h)$

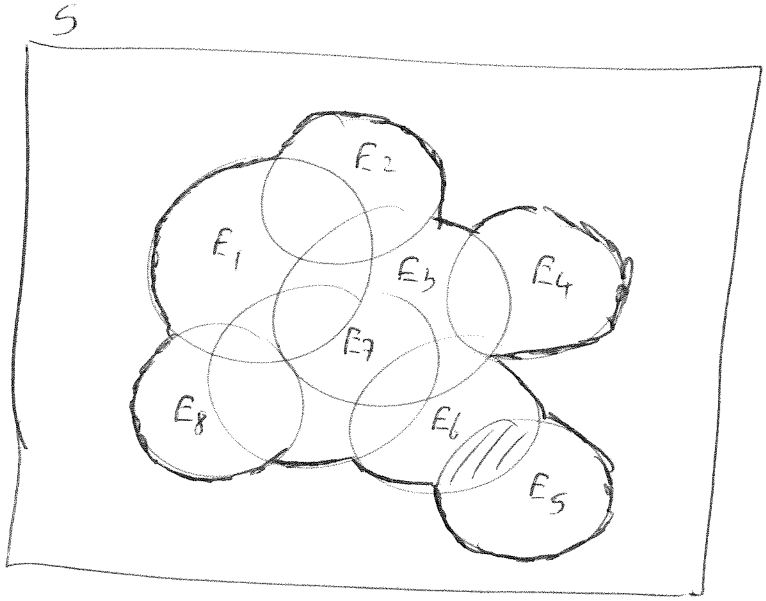
$= $ Union of everything except a

$E \cup (F \cup G) = $ " " " " "

10) If $E \subset F$, then $F = E \cup E^cF$. These are mutually exclusive, so

$$P(F) = P(E) + P(E^cF)$$

Since $P(E^cF) \geq 0$, get $P(E) \leq P(F)$

11) 

$P(\cup E_i)$ adds the probabilities of all the sample points inside the outer boundary of the circles (shown by a heavy line); all points are counted once

$\sum P(E_i)$ does the same, except some of the points may be counted multiple times (eg, points in common to E_5 and E_6 in picture above are counted twice)

So $P(\cup E_i) \leq \sum P(E_i)$

12) $P(E \cup F) = P(E) + P(F) - P(EF)$.

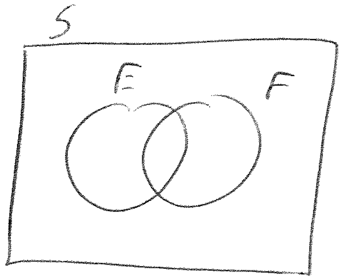
Since $P(E \cup F) \leq 1$, get

$1 \geq P(E) + P(F) - P(EF)$, or

$P(EF) \geq P(E) + P(F) - 1$

(in particular, if $P(E) = P(F) = .9$, $P(EF) \geq .9 + .9 - 1 = .8$)

13)



a) $E = EF \cup EF^c$, and these are mutually exclusive;

so $P(E) = P(EF) + P(EF^c)$, and

$$P(EF^c) = P(E) - P(EF)$$

b) $E^cF^c = (E \cup F)^c$ (De Morgan)

so $P(E^cF^c) = P((E\cup F)^c) = 1 - P(E\cup F)$

$= 1 - [P(E) + P(F) - P(EF)]$

$= 1 - P(E) - P(F) + P(EF)$

15) $\binom{9}{3} = \frac{9.8.7}{3.2.1} = 84$

$\binom{9}{6} = \frac{9.8.7.6.5.4}{6.5.4.3.2.1} = 84$

$\binom{7}{2} = \frac{7.6}{2.1} = 21$

$\binom{7}{5} = \frac{7.6.5.4.3}{5.4.3.2.1} = 21$

$\binom{10}{7} = \frac{10.9.8.7.6.5.4}{7.6.5.4.3.2.1} = 120$

16) $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n!}{(n-r)!\,r!} = \frac{n!}{(n-r)!\,(n-(n-r))!} = \binom{n}{n-r}$

OR : Choosing which r elements to put into a subset of size r from a set of size n is the same as choosing which $n-r$ elements <u>not</u> to put in.

17) $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$

$$\binom{n-1}{r-1} + \binom{n-1}{r} = \frac{(n-1)!}{(r-1)!\,(n-r)!} + \frac{(n-1)!}{r!\,(n-r-1)!}$$

$$= \frac{(n-1)!}{(r-1)!\,(n-r-1)!}\left[\frac{1}{n-r} + \frac{1}{r}\right]$$

$$= \frac{(n-1)!}{(r-1)!\,(n-r-1)!}\left[\frac{r+n-r}{r(n-r)}\right]$$

$$= \frac{(n-1)!}{(r-1)!\,(n-r-1)!}\left[\frac{n}{r(n-r)}\right]$$

$$= \frac{n!}{r!\,(n-r)!} = \binom{n}{r}$$

OR : $\binom{n}{r}$ counts # subsets of $\{1, \dots, n\}$ of size r

$\binom{n-1}{r-1}$ counts # subsets of $\{1, \dots, n\}$ of size r, that include n (once n is in, remaining $r-1$ elements must be chosen from among $n-1$)

$\binom{n-1}{r}$ counts # subsets of $\{1, \dots, n\}$ of size r, that <u>don't</u> include n (with n forbidden, all r elements must come from among $n-1$)

Since each subset of size r from $\{1, \dots, n\}$ must either include or not include n, but not both,

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$