Math 30440, Spring 2009 — Probability and Statistics

Final Examination

Wednesday, May 6, 1.45pm

Name: (please write clearly) ______________________________

Section: (circle one)

Prof. GERDES **Prof. GALVIN**

Instructions: The exam last 2 hours. There are 7 questions, equally weighted. Please show all your work. It is an open book exam.

Question	Out of	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
Total	**84**	

Good Luck!

1. A box of nails contains 6 produced by Sharp Co, 4 by Nail Inc and 2 by Hammermate Ltd (so 12 nails in all).

   (a) You select a nail at random. What's the probability that it was produced by Hammermate Ltd?

   (b) You know that nails produced by Sharp Co break on first use 1% of the time, those produced by Nail Inc break 2% of the time, those produced by Hammermate Ltd break 5% of the time. What's the probability that the nail you picked breaks on first use?

   (c) Given the information that the nail you picked does break on first use, what's the probability that is was a Hammermate Ltd nail?

2. A college gives each of the entering first years a laptop, promising to buy any working laptops back in four years. Suppose that the lifetime of such laptops is exponentially distributed with a mean lifetime of 3 years. After 2 years the college discovers that 1000 of the initial laptops are still functioning. Help the administration compute:

   (a) The probability that a particular laptop that is still working after after 2 years, will also still be working after 4 years.

   (b) The approximate probability that the college will have to buy back more than 500 laptops (assuming the lifetimes of the laptops are independent).

3. Suppose that 70% of the families in your (very large) city have no dogs, 22% have 1 dog and 8% have 2 dogs.

   (a) Let X be the number of dogs that a randomly chosen family has. Compute $E(X)$ and $Var(X)$.

   (b) Assuming your 200 family neighborhood constitutes a random sample and that families make their choices about dog ownership independently approximate the probability that their are more than 90 dogs in your neighborhood.

4. I have a coin that comes up heads with some (unknown) probability q, and I want to estimate q. I do the following experiment: I toss the coin repeatedly until I first see a head, and let T be the number of tosses it takes until this happens. I repeat this experiment n times, and get the n readings $t_1, t_2, \ldots, t_n$ for T. Use this data to give a maximum likelihood estimator for q. (Note that T has the following mass function: $P(T = k) = (1 - q)^{k-1}q$, $k = 1, 2, 3, \ldots$.)

5. Professor G. wants to estimate the proportion of students who prefer multiple-choice exams to partial credit exams. Among 100 students currently in his class, 56 prefer multiple-choice exams.

   (a) Find a 90% confidence interval for the proportion of students who prefer multiple-choice exams.

   (b) If the estimate above is to be within 0.05 of the true proportion favoring multiple-choice exams (with a 90% confidence), how many students should be sampled?

6. A person's response time to a stimulus is known to be normally distributed. I test someone's response time to the same stimulus six times, and get the following six time measurements (measured in tenths of a second):

$$8 \quad 9.5 \quad 7.8 \quad 10 \quad 9.2 \quad 9.5$$

   (a) Compute the sample mean $\bar{X}$ and sample variance S^2 of this data.

   (b) Construct a 95% confidence interval for the person's mean response time to the stimulus.

7. Is there a difference between reading or listening when it comes to retaining information? 20 randomly selected people are broken into two groups. The first group (group A, consisting of 8 people) spend a week reading factsheets about US history; the second group (group B, 12 people) listen to the same factsheets being read on an audio book. At the end of a week all 20 people take a standardized test on which the results are assumed to be normally distributed. The mean for group A was 75 with sample variance 16, and for group B was 80 with sample variance 25.

   (a) Assuming that the actual variances for the two types of groups are equal, use the data to compute S_p^2, the pooled estimator for the common variance.

   (b) Test at 5% significance the null hypothesis that the there is no difference in the mean score between readers and listeners, against the alternative that there is a difference.