# Probability and Statistics — Math 30440

# SPRING 2009

## Practice problems for the first exam

**Problem 1.**

A dodecahedron is a solid with 12 sides and is often used to display 12 months of the year. When this object is rolled, let the outcome be taken as the month appearing on the upper face. Also let $A = \{\text{January}\}$, $B = \{\text{Any months with 31 days}\}$, and $C = \{\text{Any months with exactly 30 days}\}$. Find

a) $P(A \cup B)$ b) $P(A \cap B)$ c) $P(C \cup B)$ d) $P(A \cap C)$

**Problem 2.**

The probability that $n$ electrons are emitted during a certain interval from a photoelectrically emissive surface under incident light is

$$P(n) = Cp^n \ ,$$

where $n$ can be $0, 1, 2, \ldots$ and $p$ is a positive number less than 1. Calculate the constant $C$ (it will depend on $p$). (Hint: you do not have to know anything about Physics to solve this problem.)

**Problem 3.**

Suppose 20 % of drivers on a stretch of I-90 are speeding. The speed of each driver is estimated by radar at a speed trap. The radar catches 90 % of those speeding, but it incorrectly indicates speeding for 15 % of those driving within the speed limit. What are the chances that someone caught in the speed trap is NOT guilty of speeding.

**Problem 4.**

A chip contains twenty identical transistors, which are connected in such a way that the chip will perform its function provided that no more than three of the transistors have failed. The probability that any given transistor has failed equals 0.1. Calculate the probability that the chip has failed.

**Problem 5.**

Let $F, G, H$ be **independent** events, such that $P(F) = P(G) = 0.5$ and $P(F \cup G \cup H) = 0.9$ . Find $P(H)$.

**Problem 6.**

A person has 10 friends, of whom 6 will be invited to a party.
a) How many choices are there if 2 of the friends are feuding and will not attend together?
b) How many choices if 2 of the friends will only attend together?

**Problem 7.**

One of two coins is selected at random and tossed. The first coin comes up heads with probability $p_1$ and the second coin with probability $p_2$.
a) What is the probability that the outcome of the toss is heads?
b) What is the probability that the second coin was used given that a heads occurred?

**Problem 8.**

Two cards are chosen at random from a deck of 52 playing cards. What is a probability that they have the same value (i.e., two aces, two twos ...)?

**Problem 9.**

Ten percent of items from a certain production line are defective. What is the probability that there is more than one defective item in a batch of 10 items?

**Problem 10.**

A drawer contains 6 white socks and 8 red socks. Jim selects a sock from the drawer and puts it on his left foot, then Tom selects a sock from the drawer and puts it on his right foot, then Jim selects a sock from the drawer and puts it on his right foot, then Tom selects a sock from the drawer and puts it on his left foot. What is the probability that both Jim and Tom wear matching socks, but Jim's socks are not of the same color as Tom's?

**Problem 11.**

For constants $b > 0, c > 0$ and $a$, find a condition on constant $c$ and a relationship between $c$ and $a$ (for given $b$) such that the function

$$f(x) = \begin{cases} a\left(1 - \frac{x}{b}\right) & 0 \leq x \leq c \\ 0 & \text{elsewhere} \end{cases}$$

is a valid probability density.

**Problem 12.**

A discrete random variable $X$ takes values $-1, 0, 1$. Its expected value is equal to $\frac{1}{2}$ and its variance is equal to $\frac{5}{8}$. Find the probability mass function of $X$.

**Problem 13.**
A random variable $X$ has a CDF (cumulative distribution function) given by

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 4x^3 - 3x^4 & 0 \le x < \frac{1}{2} \\ 1 & x \ge \frac{1}{2} \end{cases}$$

Find $P(X = \frac{1}{2})$.

**Problem 14.**
A gas station sells a random amount $X$ of gasoline each day. Suppose that $X$, measured in thousands of gallons, has the probability density function

$$f(x) = \begin{cases} \frac{3}{8}x^2 & 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

The owner's profit turns out to be 10 cents per gallon for the first 1000 gallons sold, and 4 cents per gallon for each additional gallon sold. Find the owner's expected profit for any given day.

**Problem 15.**
A joint probability density function of random variables $X$ and $Y$ is given by the formula

$$f(x, y) = \begin{cases} \frac{1}{4} & \text{if } |x| \le 2,\ 0 \le y \le 2 - |x| \\ 0 & \text{otherwise} \end{cases}$$

**a)** Sketch a graph showing the region of the plane for which $f(x, y)$ is non zero.
**b)** Find $P(|X| \le Y)$
**c)** Find the cumulative distribution function of the random variable $Z = X + Y$. (**Hint**: First figure out for which $a$ is $P(Z \le a) = 0$ or $P(Z \le a) = 1$; then deal with the remaining $a$).

Answers

**1.** 7/12; 1/12; 11/12; 0

**2.** $(1-p)$

**3.** 0.4

**4.** $1-5.199(0.9)^{17}$

**5.** 0.6

**6.** 136; 98

**7.** $\frac{p_1+p_2}{2}$ ; $\frac{p_2}{p_1+p_2}$

**8.** 1/17

**9.** $1-1.9(0.9)^9$

**10.** 20/143

**11.** $c < b$ ; $a = \frac{2b}{c(2b-c)}$

**12.** $p(-1) = 3/16$ , $p(0) = 1/8$ , $p(1) = 11/16$

**13.** 11/16

**14.** 180.125

**15.** $f$ is non-zero in the triangle with vertices $(-2,0),(2,0)$ and $(0,2)$; $\frac{1}{2}$; $F_Z(a) = 0$ for $a < -2$ and 1 for $a \geq 2$. For the remaining $a$, $F_Z(a)\frac{(a+2)^2}{16}$.