The diet problem — sensitivity analysis

Two available brands of cereal:

Krunchies, costing 3.8 cents per ounce

Crispies, costing 6.2 cents per ounce

Breakfast nutrition requirements:

Thiamine: at least 1 mg

Niacin: at least 5 mg

Energy: at least 900 calories, at most 1500

Nutritional info for Krunchies and Crispies (per ounce):

	Thiamine	Niacin	Energy
Krunchies:	.1	1	110
Crispies:	.25	.25	120

The problem:

Produce a low-cost breakfast that satisfies nutritional requirements

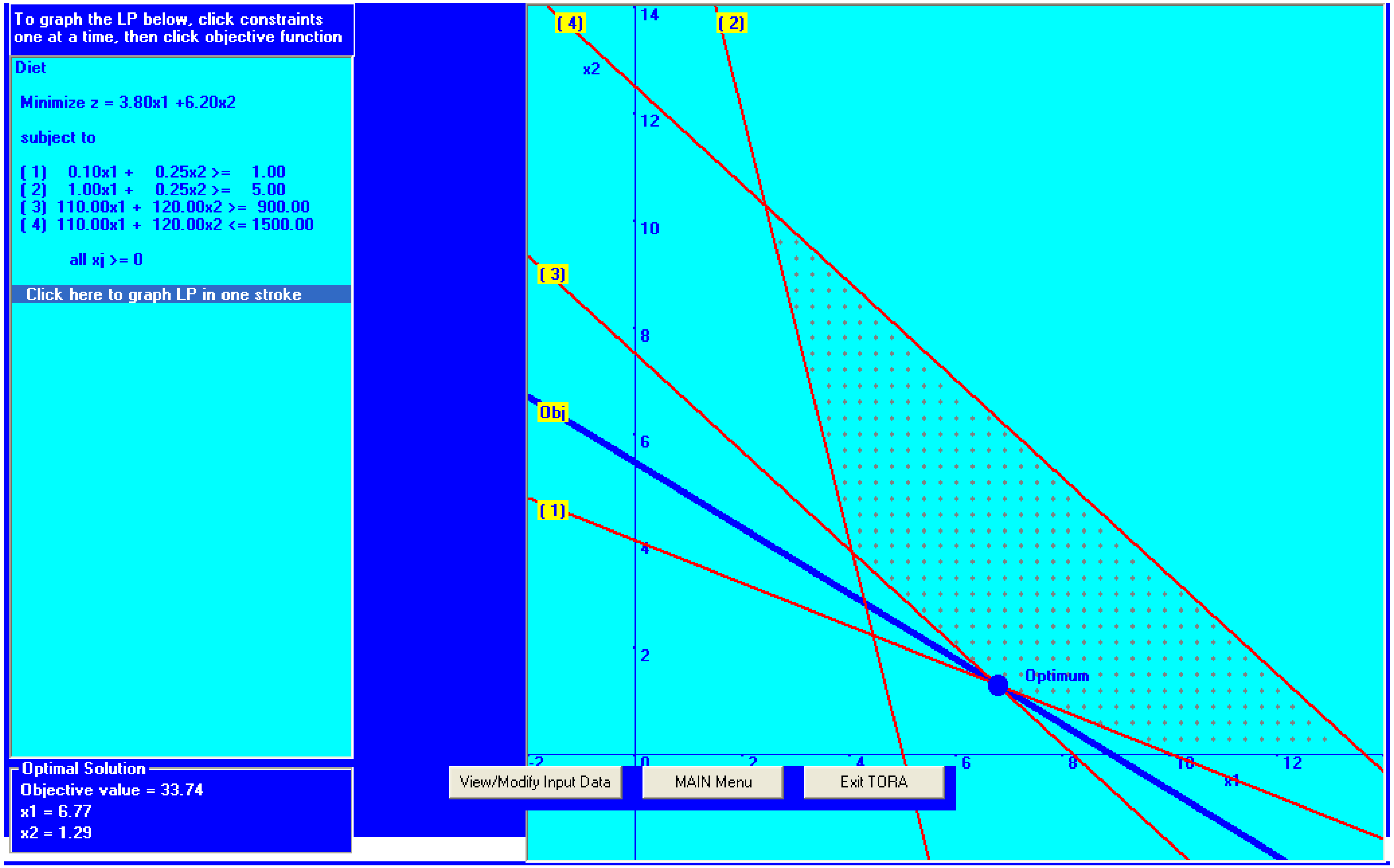
The Linear Programming formulation

K = number of ounces of Krunchies

C = number of ounces of Crispies

$$
\begin{array}{lcr}
\text{Minimize} & 3.8K + 6.2C & \text{(total cost)} \\
\text{Subject to} & .1K + .25C \geq 1 & \text{(thiamine need)} \\
 & K + .25C \geq 5 & \text{(niacin need)} \\
 & 110K + 120C \geq 900 & \text{(energy need)} \\
 & 110K + 120C \leq 1500 & \text{(energy restriction)} \\
 & K \geq 0,\ C \geq 0 &
\end{array}
$$

Solution via TORA



$K = 6.77,\ C = 1.29$; cost 33.74 cents

Initial and final tableaus ($M = 50$)

Iteration 1	K	C								
Basic	x1	x2	Sx3	Sx4	Sx5	Rx6	Rx7	Rx8	sx9	Solution
z (min)	5551.2000	6018.8000	-50.0000	-50.0000	-50.0000	0.0000	0.0000	0.0000	0.0000	45300.0000
Rx6	0.1000	0.2500	-1.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000
Rx7	1.0000	0.2500	0.0000	-1.0000	0.0000	0.0000	1.0000	0.0000	0.0000	5.0000
Rx8	110.0000	120.0000	0.0000	0.0000	-1.0000	0.0000	0.0000	1.0000	0.0000	900.0000
sx9	110.0000	120.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1500.0000

Iteration 5	K	C								
Basic	x1	x2	Sx3	Sx4	Sx5	Rx6	Rx7	Rx8	sx9	Solution
z (min)	0.0000	0.0000	-14.5806	0.0000	-0.0213	-35.4194	-50.0000	-49.9787	0.0000	33.7419
x2	0.0000	1.0000	-7.0968	0.0000	0.0065	7.0968	0.0000	-0.0065	0.0000	1.2903
x1	1.0000	0.0000	7.7419	0.0000	-0.0161	-7.7419	0.0000	0.0161	0.0000	6.7742
Sx4	0.0000	0.0000	5.9677	1.0000	-0.0145	-5.9677	-1.0000	0.0145	0.0000	2.0968
sx9	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	-1.0000	1.0000	600.0000

Dual price for Thiamine constraint: 14.58

Dual price for Niacin constraint: 0 (optimum provides $5 + 2.1$ mgs of Niacin; changing Niacin demand slightly won't move optimum)

Dual price for minimum calorie constraint: 0.021

Dual price for maximum calorie constraint: 0 (as with the Niacin constraint, the max. calorie constraint is not met tightly)

Feasible ranges for changes to right-hand side

If: Thiamine demand changes from 1 to $1 + \Delta_1$

Niacin demand changes from 5 to $5 + \Delta_2$

Minimum calorie requirement changes from 900 to $900 + \Delta_3$

Maximum calorie requirement changes from 1500 to $1500 + \Delta_4$

then: Minimum cost changes to $33.74 + 14.58\Delta_1 + .021\Delta_3$

Optimum value for K changes to $6.77 - 7.74\Delta_1 + .016\Delta_3$

Optimum value for C changes to $1.29 + 7.1\Delta_1 - .0065\Delta_3$

as long as: $1.29 + 7.1\Delta_1 - .0065\Delta_3 \geq 0$

$6.77 - 7.74\Delta_1 + .016\Delta_3 \geq 0$

$2.1 - 5.97\Delta_1 - \Delta_2 + .015\Delta_3 \geq 0$

$600 - \Delta_3 + \Delta_4 \geq 0$

or individually: $-.18 \leq \Delta_1 \leq .35$ $\quad -\infty \leq \Delta_2 \leq 2.1$

$-140 \leq \Delta_3 \leq 198$ $\quad -600 \leq \Delta_4 \leq \infty$

Example

If: Thiamine demand remains at 1

Niacin demand changes from 5 to 4

Minimum calorie requirement changes from 900 to 800

Maximum calorie requirement changes from 1500 to 1000

then: $\Delta_1 = 0, \quad \Delta_2 = -1, \quad \Delta_3 = -100, \quad \Delta_4 = -500$

and: Cost changes to $33.74 + 14.58 * 0 - .021 * 100 = 31.64$

K changes to $6.77 - 7.74 * 0 - .016 * 100 = 5.17$

C changes to $1.29 + 7.1 * 0 + .0065 * 100 = 1.94$

because: $1.29 + 7.1 * 0 + .0065 * 100 \geq 0$

$6.77 - 7.74 * 0 - .016 * 100 \geq 0$

$2.1 - 5.97 * 0 + 1 - .015 * 100 \geq 0$

$600 + 100 - 500 \geq 0$