Math 30210 — Introduction to Operations Research

Quiz 1 – Wednesday September 5, 2007

NAME: _____

Instructions: This is a closed-book quiz. Please do not use any notes.

A caterer has five mixed fruit drinks available to him, and must produce 500 gallons of punch for a party. The host requires that the punch must contain at least 20% orange juice, at least 10% grapefruit juice and at least 5% cranberry juice. The inventory data are as shown below. The caterer wants to obtain the minimum-cost blend that meets these requirements. Formulate this problem as a linear program.

	Orange	Grapefruit	Cranberry	Supply	Cost
Drink 1	40%	40%	0%	200 gal	$1.5
Drink 2	5%	10%	20%	400 gal	$.75
Drink 3	100%	0%	0%	100 gal	$2
Drink 4	0%	100%	0%	50 gal	$1.75
Drink 5	0%	0%	0%	800 gal	$.25

Solution:
Let x_i be number of gallons of Drink i used ($i = 1, 2, 3, 4, 5$).
Minimize
$$1.5x_1 + .75x_2 + 2x_3 + 1.75x_4 + .25x_5 \quad \text{(cost)}$$
subject to

$$
\begin{aligned}
x_1 + x_2 + x_3 + x_4 + x_5 &= 500 \quad \text{(demand constraint)} \\
.4x_1 + .05x_2 + x_3 &\geq 100 \quad \text{(orange juice constraint)} \\
.4x_1 + .1x_2 + x_4 &\geq 50 \quad \text{(grapefruit juice constraint)} \\
.2x_2 &\geq 25 \quad \text{(cranberry juice constraint)},
\end{aligned}
$$

all $x_i \geq 0$, and the inventory constraints

$$x_1 \leq 200, \; x_2 \leq 400, \; x_3 \leq 100, \; x_4 \leq 50, \; x_5 \leq 800.$$

(**Solution**: $x_1 = 93.75$, $x_2 = 125$, $x_3 = 56.25$, $x_4 = 0$ and $x_5 = 225$.)