Math 30210 — Introduction to Operations Research

Assignment 9 (55 points total)

Due before class, Wednesday November 7, 2007

Instructions: Please present your answers neatly and legibly. Include a cover page with your name, the course number, the assignment number and the due date. The course grader reserves the right to leave ungraded any assignment that is disorganized, untidy or incoherent. You may turn this assignment in before class, or leave it in my mailbox (outside 255 Hurley Hall). It can also be emailed; if you plan to email, please check with me to see if the format you plan to use is one that I can read. No late assignments will be accepted. It is permissible (and encouraged) to discuss the assignments with your colleagues; but the writing of each assignment must be done on your own.

Reading: Sections 4.2 and 4.3.

1. (8 points) A primal problem (given in standard form) is:
   Maximize $c^t x$ subject to $Ax = b$, $x \geq 0$ (where c and x are n-dimensional column vectors, b is an m-dimensional column vector and A is an m by n matrix).

   (a) Write down the dual problem.

   (b) Show that if x is a feasible vector for the primal problem, and y is a feasible vector for the dual problem, then $c^t x \leq b^t y$ (i.e., all feasible primal solutions are less than or equal to all feasible dual solutions).

   (c) Show that if the primal problem is a minimization problem, then all feasible primal solutions are greater than or equal to all feasible dual solutions.

2. (6 points) For both of these parts, use the result of the previous question.

   (a) If the primal problem is a maximization, and is unbounded, what can you say about the dual problem? Justify.

   (b) If the primal problem is a maximization, and there is no feasible solution, what can you say about the dual problem? Justify.

3. (8 points) Consider the following linear programming problem:
Minimize $x_1 - x_2 + 3x_3$ subject to
$-x_1 + x_3 \leq 4$
$-2x_1 - 3x_2 + 3x_3 \geq 5$
x_1 unrestricted, $x_2, x_3 \geq 0$.

   (a) Convert the problem to standard form, and then find the dual using the definition of the dual given in Taha pages 152-153.

   (b) Write down the dual directly (without converting to standard form) using either Taha Table 4.3 (page 155) or the "Primal-dual dictionary" on the course website.

   (c) Show that the two dual problems you have constructed are the same.

4. (5 points) Taha 4.2C problem 1

5. (6 points) Taha 4.2C problem 4. (On page 159, Taha describes two methods for finding the dual solution, which he refers to as Method 1 and Method 2. I only talked about Method 2 in class; if you wish, you may just solve this problem using Method 2).

6. (5 points) Taha 4.2C problem 8, part (b).

   For each of the previous six problems, you should not need to use (and should not use) TORA. For the remaining three problems, TORA may be helpful.

7. (5 points) Taha 4.2D problem 1.

8. (6 points) Taha 4.3A problem 3.

9. (6 points) Taha 4.3B problem 3.