Math 30210 — Introduction to Operations Research

Assignment 4 (75 points total)

Due before class, Wednesday September 26, 2007

Instructions: Please present your answers neatly and legibly. Include a cover page with your name, the course number, the assignment number and the due date. The course grader reserves the right to leave ungraded any assignment that is disorganized, untidy or incoherent. You may turn this assignment in before class, or leave it in my mailbox (outside 255 Hurley Hall). It can also be emailed; if you plan to email, please check with me to see if the format you plan to use is one that I can read. No late assignments will be accepted. It is permissible (and encouraged) to discuss the assignments with your colleagues; but the writing of each assignment must be done on your own.

Reading: Sections 3.3.2, 3.3.3 and 3.4.1.

1. (12 points) Taha 3.3B Problem 2 (a), b) and d) only).

   Note: You should begin by doing each of the above problems *by hand*. Set up the initial tableau by hand; identify the initial entering and departing basic variables; do the pivoting to reach the second tableau by hand; identify the second entering and departing basic variables, and do the pivoting to reach the third tableau by hand, etc. You should also complete the problem using TORA, and submit a copy of the TORA printout. Note that your tableau should agree with TORA's, and you should certainly use TORA to verify that you have done the tableau correctly, but you should do the problem *by hand* before turning to TORA for verification. See my worked solution of part c) for a guide on how I expect the solutions to these problems to be presented.

2. (6 points) Taha 3.3B Problem 3 (You must do this problem by hand).

3. (6 points) Taha 3.3B Problem 4.

4. (8 points) Taha 3.3B Problem 6 (You must do this problem by hand).

5. (8 points) Taha 3.3B Problem 8 (You must do this problem by hand).

6. (4 points) Taha 3.3B Problem 12.

7. (5 points) Taha 3.4A Problem 2.

8. (8 points) Taha 3.4A Problem 3 (a) and b) only). (You should do this problem by hand).

9. (8 points) Taha 3.4A Problem 4 (a) and b) only). (You should do this problem by hand).

10. (5 points) Taha 3.4A Problem 6 (You should do this problem by hand).

11. (5 points) Taha 3.4A Problem 9 (You should do this problem by hand).

Solution to Taha 3.3B Problem 2c):

After introducing slack variables x_5, x_6 and x_7, all non-negative, for constraints one, two and three respectively, the initial tableau is

basic	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	solution
	1	-3	1	-3	-4	0	0	0	0
x_5	0	1	2	2	4	1	0	0	40
x_6	0	2	-1	1	**2**	0	1	0	8
x_7	0	4	-2	1	-1	0	0	1	10

The most negative entry in the z-row is the -4 corresponding to x_4, so this will be our entering variable. Comparing the ratios $40/4$, $8/2$ and $10/-1$, we find that $8/2$ is the smallest positive ratio, so x_6 will be the first departing variable. After pivoting on the pivot element, the bolded **2** above, the new tableau is

basic	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	solution
	1	1	-1	-1	0	0	2	0	16
x_5	0	-3	**4**	0	0	1	-2	0	24
x_4	0	1	$-.5$	.5	1	0	.5	0	4
x_7	0	5	-2.5	1.5	0	0	.5	1	14

There are two negative entries in the z-row now; since they are equal, we by convention choose the leftmost of them and take x_2 as our new entering variable. Our departing variable is x_5, since it is the only one whose coefficient ratio with the entry in the solution column is positive. So we pivot on the bolded **4** to reach the third tableau:

basic	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	solution
	1	.25	0	-1	0	.25	1.5	0	22
x_2	0	$-.75$	1	0	0	.25	$-.5$	0	6
x_4	0	.63	0	**.5**	1	.13	.25	0	7
x_7	0	3.13	0	1.5	0	.63	$-.75$	1	29

This is still not optimal, since there is a negative entry on the z-row. So we should next pivot on the bolded **.5**, to bring x_3 into the collection of basic variables, and remove x_4.

The next tableau is reported below.

basic	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	solution
	1	1.5	0	0	2	.5	2	0	36
x_2	0	$-.75$	1	0	0	.25	$-.5$	0	6
x_3	0	1.25	0	1	2	.25	.5	0	14
x_7	0	1.25	0	0	-3	.25	-1.5	1	8.

Now all of the entries in the z-row are positive, so we cannot increase the objective any more by introducing a new basic variable. We have reached an optimum: $x_2 = 6, x_3 = 14, x_1 = 0, z = 36$.