Math 30210 — Introduction to Operations Research

Assignment 1 (50 points total)

Due before class, Wednesday September 5, 2007

Instructions: Please present your answers neatly and legibly. Include a cover page with your name, the course number, the assignment number and the due date. The course grader reserves the right to leave ungraded any assignment that is disorganized, untidy or incoherent. You may turn this assignment in before class, or leave it in my mailbox (outside 255 Hurley Hall). It can also be emailed; if you plan to email, please check with me to see if the format you plan to use is one that I can read. No late assignments will be accepted. It is permissible (and encouraged) to discuss the assignments with your colleagues; but the writing of each assignment must be done on your own.

Reading: Chapter 1, and Sections 2.1, 2.2 and 2.3

1. (2 points) Taha, Problem Set 1.1A, problem 1.
2. (2 points) Addition to the previous problem: identify a feasible alternative (other than Alternative 3) that is also optimal.
3. (5 points) Taha, Problem Set 1.1A, problem 4.
4. (17 points total) A more general version of the previous problem:

   Four friends are gathered on one side of a river. (Their names are Stuhldreher, Miller, Crowley and Layden, but we will call them F_1, F_2, F_3 and F_4.)

   They want to cross to the other side of the river, but they only have one rowboat which can carry a maximum of two people at one time. F_1 can row across the river in a_1 minutes, F_2 in a_2 minutes, etc. For the sake of convenience, we has listed the friends in such a way that $a_1 \le a_2 \le a_3 \le a_4$.

   If two people are in the boat, the time taken to cross is that of the slower of the two rowers (e.g., if F_1 and F_2 row together, the journey will take a_2 minutes).

   The rowboat cannot cross the river without a rower in it; also, it is an old rowboat, and can only manage a total of five one-way journeys before it sinks.

   (a) (2 points) How many feasible schemes are there to get the friends across the river?

(b) (2 points) Describe a scheme which you suspect minimizes the time taken for the friends to cross the river. How long does it take?

(c) (7 points) Prove that the scheme you have described is indeed the best. (In the course of answering this part, you may discover that your originally proposed scheme is *not* the best, or is only the best for certain values of a_1, a_2, a_3 and a_4, in which case you should start over...)

(d) Critique the model presented in this problem; specifically:

   i. (1 point) Do you think that this is a realistic model?

   ii. (2 point) Are there any factors that model ignores?

   iii. (3 points) Can you propose what you think might be a better model?

5. (Optional!) Taha, Problem Set 1.1A, problem 6.

6. (2 points) Consider the first case described in Taha, Section 1.5 (the case involving the elevators). Critique the proposed solution. Specifically, do you think it satisfactorily resolves the problem?

7. (Optional!) Taha, Problem Set 2.1A, problems 1-4.

   Note: for the next two items, you should set up the problem as a linear programming problem, assigning appropriate variables, identifying the objective function, and identifying all the constraints. Then you should solve the problem graphically. For the first problem (Taha 2.2A. problem 4) you **must** draw the graphs by hand. For the second and third, you may if you wish print out a TORA screenshot.

8. (10 points total) Taha, Problem Set 2.2A, problems 6 (5 points), 15 (5 points) and (Optional!) 16.

9. (5 points total) Taha, Problem Set 2.2B, problems 4 (5 points) and (Optional!) 7.

10. (7 points total) A furniture maker has 6 units of wood and 28 hours of free time, in which he will make decorative screens. Two models have sold well in the past, so he will restrict himself to those two. He estimates that model I requires 2 units of wood and 7 units of time, while model II requires 1 unit of wood and 8 hours of time. The prices for the models are \$ 120 and \$ 80 respectively. The furniture maker wishes to maximize his sales revenue.

   (a) (2 points) Formulate this as a mathematical problem: assign appropriate variables, identify the objective function, and identify all the constraints.

   (b) (1 point) Is the problem you formulated in the first part a linear programming problem of the type presented in Taha, Problem Sets 2.2A and 2.2B? If not, why not?

   (c) (4 points) Solve the problem and identify how many screens of each model the furniture maker should make in order to maximize his profit.