Statistics for the Life Sciences

Math 20340 Section 01, Fall 2009

Homework 2 Solutions

- **4.37**:

  The experiment here is to pick one of the possible ways in which the board votes 5-3 in favour. To construct a simple event, we need just selected the five people who voted in favour of the plaintiff; the unchosen three automatically become the three who voted against. There are $C_5^8 = 56$ ways to do this (note that we don't care about the order in which people voted), and so 56 simple events. In only one of those 56 do all 5 women vote in favour of plaintiff; so the probsbility of seeing this split is $1/56$.

- **4.38**:

  There are $C_5^{10} = 252$ ways for the instructor to choose 5 questions (order doesn't matter). Of these 252, only $C_5^6 = 6$ include only questions that the student has prepared. So he probability is $6/252 \approx .0238$.

- **4.42**:

  - **c**: $B \cap C = \{E_4\}$, so $P(B \cap C) = .2$
  - **g**: $A \cup B \cup C = \{E_1, E_2, E_3, E_4, E_5\}$, so $P(A \cup B \cup C) = 1$
  - **h**: $A \cap B = \{E_1\}$ so $(A \cap B)^c = \{E_2, E_3, E_4, E_5\}$ and $P((A \cap B)^c) = 4/5$

- **4.43**:

  - **b**: $P((A \cap B)^c) = 1 - P(A \cap B) = 1 - .2 = .8\ (= 4/5)$

- **4.44**:

  - **a**: $P(A|B) = P(A \cap B)/P(B) = .2/.8 = .25\ (= 1/4)$

- **4.45**:

  - **c**: $P(B \cup C) = P(B) + P(C) - P(B \cap C)$ so $P(B \cap C) = P(B) + P(C) - P(B \cup C) = .8 + .4 - 1 = .2$

- **4.46**:

  - **a**: $P(A|B) = .25$ and $P(A) = .4$, so the events are NOT independent

- **b**: $A \cap B = \{E_1\}$ which is not the empty set, so the events are NOT mutually exclusive

- **4.50**:
  - **a**: Since the events are mutually exclusive, the probability of the intersection is 0
  - **b**: Since the events are mutually exclusive, the probability of the union is the sum of the probabilities, so $P(A \cup B) = .3 + .5 = .8$

- **4.52**:
  - **a**: $.49$
  - **b**: $.8$
  - **c**: $.34$
  - **d**: $.95$
  - **e**: $.34/.8 = 17/40$
  - **f**: $.34/.49 = 34/49$

- **4.56**:
  - **a**: $.4$
  - **b**: $.37$
  - **c**: $.1$
  - **d**: $.67$
  - **e**: $.6$
  - **f**: $.33$
  - **g**: $.9$
  - **h**: $.1/.37 \approx .27$
  - **i**: $.1/.4 = .25$

- **4.60**:
  - **a**: $S = \{\text{Starbucks}\}, M = \{\text{Mocha}\}$. We want $P(S \cap M)$. Since S and M are given to be independent, $P(S \cap M) = P(S)P(M) = .7X.6 = .42$
  - **b**: Yes, it is given in the question that choice of drink is not influenced by choice of coffee shop
  - **c**: $P = \{\text{Peets}\}$. Want $P(P|M) = P(P \cap M)/P(M) = (.3 * .6)/.6 = .3$. Or, easier: $P(P|M) = P(P) = .3$ since P and M are independent
  - **d**: $P(S \cup M) = P(S) + P(M) - P(S \cap M) = .7 + .6 - .42 = .88$

- **4.62**:

  Let S be the event that a randomly chosen person is a smoker, and L the event that a randomly chosen person dies from lung cancer. We are given:

- $P(S) = .2$ (so $P(S^c) = .8$)
- $P(L|S) \approx 10P(L|S^c)$
- $P(L) = .006$

A person who dies of lung cancer either smokes or doesn't (but not both), so $P(L) = P(L \cap S) + P(L \cap S^c)$. Since $P(L \cap S) = P(S)P(L|S) = .2P(L|S)$ and $P(L \cap S^c) = P(S^c)P(L|S^c) = .8P(L|S^c)$ we get $.006 = .2P(L|S) + .8P(L|S^c)$. Solving this simultaneously with $P(L|S) \approx 10P(L|S^c)$ we get $.006 \approx 2P(L|S^c) + .8P(L|S^c)$ or $P(L|S^c) \approx .00214$ and so $P(L|S) \approx .0214$.

- **4.65**:
  - **d**: $88/154$
  - **e**: $44/67$
  - **f**: $23/35$
- **4.67**:
  - **a**: $.8X.8 = .64$ (throws are independent)
  - **b**: Probability that Shaq makes only first of two: $.53X.47 = .2491$. Probability that Shaq makes only second of two: $.47X.53 = .2491$. These are the two mutually exclusive events that combine to the event of Shaq making exactly one; so probability that Shaq makes exactly one: $.2491 + .2491 = .4982$.
  - **c**: $A = \{$Shaq makes both$\}$. $B = \{$Jason makes neither$\}$. $P(A) = .53 * .53 = .2809$. $P(B) = .2 * .2 = .04$. Since A and B are independent, $P(A \cap B) = P(A)P(B) = .011246$
- **4.75**:

  Given:
  - $P(L) = .3$, $P(R) = .7$
  - $P(S|R) = .8$, $P(B|R) = .2$
  - $P(S|L) = .1$, $P(B|L) = .9$

  From this we get
  - $P(S \cap R) = P(R)P(S|R) = .7 * .8 = .56$
  - $P(S \cap L) = P(L)P(S|L) = .3 * .1 = .03$
  - $P(B \cap R) = P(R)P(B|R) = .7 * .2 = .14$
  - $P(B \cap L) = P(L)P(B|L) = .3 * .9 = .27$

  and so
  - $P(S) = P(S \cap R) + P(S \cap L) = .56 + .03 = .59$

- $P(B) = P(B \cap R) + P(B \cap L) = .14 + .27 = .41$

- **a**: We want to know the probability that the play goes to the left *GIVEN* that a balanced stance is observed. $P(L|B) = P(L \cap B)/P(B) = .27/.41 = .66...$
- **b**: $P(R|B) = P(R \cap B)/P(B) = .14/.41 = .34...$
- **c**: On seeing balanced stance, you know that with probability .34 the play will go to the right, and with probability .66 it will go to the left. This suggests it is better to prepare for a play going to the left.

• **4.78**:

Let D be the event that the person confronted denies, and G the event that they are guilty. We want $P(G|D) = P(G \cap D)/P(D)$. Since 80% of the 5% who are guilty deny, the probability of $G \cap D$ is $.04$. The probability of D is $.06$ — $.04$ contribution from those who are guilty, .02 contribution from those who unknowingly made a mistake (I assume that everyone who does not realize that they made a mistake will deny that they have when confronted), and no contribution from those who made no mistake (they will never be confronted). So $P(G|D) = .04/.06 = 2/3$