Math 20340 — Statistics for Life Sciences

Fall 2009 third mid-term exam

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This examination contains 6 problems on 7 pages (including the front cover). The exam also comes with a standard normal distribution table, a t-table and a table of useful formulae. It is closed-book, but you may use up to four single sided pages of handwritten notes. You may use a calculator. **Show all your work** on the paper provided. The honor code is in effect for this examination.

Scores

Question	Score	Out of
1		10
2		10
3		10
4		10
5		10
6		10
Total		60

GOOD LUCK !!!

1. Does alcohol slow reaction times? To test a research hypothesis that after consuming alcohol, people generally have slower reaction times, seven randomly chosen subjects have their reaction times to a stimulus measured. The seven subjects are then treated to a three ounce glass of whiskey, and after a short while their reaction times to the same stimulus are measured. The data is as follows (reaction times measured in tenths of seconds):

Before	4	5	5	4	3	6	2
After	8	8	4	6	5	6	6.

(a) Does this data support the research hypothesis at 5% significance?

Use paired difference test $H_0 : \mu_d = 0$

$H_1 : \mu_d > 0$

$\bar{X}_d = 2, \; S_d^2 = \frac{11}{3}, \; S_d = 1.91\ldots$

$$\frac{\bar{X}_d}{S_d/\sqrt{7}} = 2.76\ldots$$

$$\left.\begin{array}{l} t_{.025} = 2.447 \\ t_{.010} = 3.143 \end{array}\right\} df = 6$$

So $.01 < p < .025$, accept H_1 at 5%.

(b) Does the data support the research hypothesis at 10% significance?

Since $p < .025$, accept at 10% also

2. An experiment was conducted to compare the mean absorption times for two drugs, A and B. Ten people were randomly selected, and split into two equal groups. One group received drug A, and the average time to absorption into the bloodstream for this group was 27.2 minutes, with sample variance 16.36. The other group received drug B, and the average time to absorption was 33.5 minutes, with sample variance 18.92.

   (a) What assumptions do you need to make about the distributions and variances of the two populations in order to be able to run a small-sample difference of means test?

   - Both populations normally distributed
   - Population variances are equal (or close)

   (b) Does the data provide evidence, at 5% significance, that the mean absorption times for the two drugs are different?

$H_0 : \mu_A = \mu_B$ $\quad \bar{x}_A = 27.2 \quad s_A^2 = 16.36$

$H_1 : \mu_A \neq \mu_B$ $\quad \bar{x}_B = 33.5 \quad s_B^2 = 18.92$

$$s^2 = \frac{4 \times 16.36 + 4 \times 18.92}{8}$$

$$= 17.64$$

$$\frac{\bar{x}_B - \bar{x}_A}{\sqrt{\frac{s^2}{5} + \frac{s^2}{5}}} = 2.37$$

$t_{.025} = 2.306$ ($df = 8$)

So p-value > .05, enough evidence to reject null.

3. Random samples of 200 bolts manufactured by machine A and 150 bolts manufactured by machine B had 16 and 6 defectives, respectively.

(a) If the two machines A, B have on average the same proportion of defectives, what is the best estimate that you can give of that proportion, using the given the data?

$$\hat{p} = \frac{16+6}{200+150} = .063\ldots$$

[Pooled estimator]

(b) Is there evidence from this data to conclude, at 10% significance, that machine A produces a higher proportions of defectives than machine B?

$H_0 : p_A = p_B$

$H_1 : p_A > p_B$

$\hat{p}_A = .08, \quad \hat{p}_B = .04$

$$\frac{\hat{p}_A - \hat{p}_B}{\sqrt{\frac{\hat{p}\hat{q}}{200} + \frac{\hat{p}\hat{q}}{150}}} = 1.538 \qquad (\hat{p} = .063\ldots)$$

Critical value for accepting H_1 at 10% : 1.28

Since $1.538 > 1.28$, there is evidence to conclude that $p_A > p_B$.

4. An experimenter is interested in determining the mean thickness of the cortex of the sea urchin egg. She measures the thickness for five randomly selected urchins and gets the following measurements:

4.5 4.7 4.6 4.1 4.8.

(a) Compute $\bar{x}$ and s for this data.

$$\bar{x} = 4.54$$

$$s = .270\ldots$$

(b) Assuming a normal distribution of thicknesses, write down a 98% confidence interval for the mean thickness.

$df = 4$

98% Confidence interval:

$$\left(\bar{x} - t_{.01} \frac{s}{\sqrt{n}}, \ \bar{x} + t_{.01} \frac{s}{\sqrt{n}} \right)$$

$$= \left(4.54 - 3.747 \frac{.27}{\sqrt{5}}, \ 4.54 + 3.747 \frac{.27}{\sqrt{5}} \right)$$

$$= (4.0872\ldots, \ 4.9928\ldots)$$

5. I want to test whether there is a difference between two population means, so I take some random samples. From the first population I sample 40 times, and get a sample mean of 2980 and sample standard deviation of 1140. From the second population I also sample 40 times, and get a sample mean of 3205 and sample standard deviation of 963.

   (a) Is there enough evidence from this data to conclude, at 5% significance, that the populations have different means?

$H_0 : \mu_1 = \mu_2$ $\quad \bar{x}_1 = 2980$ $\quad \bar{x}_2 = 3205$

$H_1 : \mu_1 \neq \mu_2$ $\quad s_1 = 1140$ $\quad s_2 = 963$

$n_1 = 40$ $\quad n_2 = 40$

$$\frac{\bar{x}_2 - \bar{x}_1}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = .95\ldots$$

Critical value for rejecting null at 5% is 1.96

Since $.95 \not> 1.96$, $.95 \not< -1.96$, do not reject null

   (b) Compute the p-value for this test.

p-value $= P(z \geq .95 \text{ or } \leq -.95)$

$= .34$.

6. Ecologists generally believe that a certain type of tropical woodland contains on average 35 kg of biomass per square meter. Recent data collection may suggest that this figure could be too high. Specifically, a recent study was made of [redacted] 100 randomly chosen square meters of tropical woodland. The average biomass per square meter was calculated to be $\bar{x}$. The sample standard deviation was 10. Letting μ be the actual average biomass per meter square, we want to use this data to test $H_0 : \mu = 35$ against the research alternative $H_a : \mu < 35$, at 5% significance.

   (a) What is the range of values of $\bar{x}$ that would lead to acceptance of the null hypothesis for this test?

   Accept null if $\dfrac{\bar{x} - 35}{10/\sqrt{100}} \geqslant -1.645$

   $\bar{x} \geqslant 33.355$

   (b) Suppose that in fact μ is 32 kg per square meter. Compute the value of β for this test.

   $\beta = P(\bar{x} \geqslant 33.355)$ when $\bar{x}$ is Normal, $\mu = 32$, $\sigma = 10/\sqrt{100}$

   $= P\left(z \geqslant \dfrac{33.355 - 32}{10/\sqrt{100}}\right)$

   $= P(z \geqslant 1.355)$

   $= .0869$

   (c) Answer the following questions for this test (no need for long justifications).

      i. What would happen to the value of β if the sample size was decreased?

      β would ~~decrease~~ increase (Smaller sample, more chance of error)

      ii. What is the probability that the null hypothesis will be rejected when in fact it is true?

      5% (.05, the significance)

      iii. What is the probability that the null hypothesis will be rejected if the true mean is in fact 32?

      This is the power, $1-\beta$, so (by b))

      .9131