Math 20340 — Statistics for Life Sciences

Fall 2009 second mid-term exam

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This examination contains 6 problems on 8 pages (including the front cover and a table of useful formulae at the end). The exam also comes with a standard normal distribution table. It is closed-book, but you may use up to four single sided pages of handwritten notes. You may use a calculator. **Show all your work** on the paper provided. The honor code is in effect for this examination.

Scores

Question	Score	Out of
1		10
2		10
3		10
4		10
5		10
6		10
Total		60

GOOD LUCK !!!

1. Answer these questions using the standard normal table included with this exam. In the first two parts, z is a standard normal.

(a) Find a z_0 such that the probability that z is greater than z_0 is .281.

From table, $P(z \le .58) = .719$,

So $P(z \geq .58) = 1 - .719 = .281$

$z_0 = .58$

(b) What is the probability that z is either greater than 1.2 or smaller than -1.2? (Your answer should be a single number)

$P(z \geq 1.2) = 1 - .8849 = .1151$

$P(z \le -1.2) = .1151$

So $P(z \geq 1.2 \text{ or } z \le -1.2) = 2(.1151)$

$= .2302$

(c) A certain normal random variable x is known to have mean 0, but its standard deviation is unknown. If data suggests that the probability that x is greater than 5 is around .2, find the standard deviation.

If x has std dev σ, $\frac{x}{\sigma} \approx z$

$P(z \geq .84) \approx .2$,

So $P(\frac{x}{\sigma} \geq .84) \approx .2$,

$P(x \geq .84\sigma) \approx .2$

But $P(x \geq 5) \approx .2$, So $.84\sigma = 5$,

$\sigma \approx \frac{5}{.84} = 5.917$

2. People's reaction to a certain stimulus is known to be normally distributed with mean 1.08 seconds, variance .4.

   (a) What is the probability that a randomly chosen subject will react to the stimulus in less than 1 second?

X = reaction time , $X \approx \text{Normal}(1.08, .4)$

$$P(X \leq 1) = P\left(\frac{X - 1.08}{\sqrt{.4}} \leq \frac{1 - 1.08}{\sqrt{.4}}\right)$$

$$= P(Z \leq -.126)$$

$$\approx .4483$$

   (b) If three subjects are selected at random, what is the probability that at least two of them will react to the stimulus in less than 1 second?

This is binomial trial , $n = 3$, $p = .4483$.

Want $P(\geq 2) = C^3_2 \, p^2 (1-p) + C^3_3 \, p^3$

$$= .422\ldots$$

3. An entomologist wishes to estimate the average development time of the citrus red mite, correct to within .5 days. From previous observations, it is known that the development times typically range from 48 to 64 days.

(a) Give an estimate for σ, the standard deviation of the development time of a randomly chosen citrus red mite.

$$\sigma \approx \frac{\text{Range}}{4} = \frac{64-48}{4} = \frac{16}{4} = 4$$

(b) What sample size is needed to be 95% sure that the average development time has been estimated to within $\pm .5$ days?

Want n so that $\pm 1.96 \frac{4}{\sqrt{n}} = \pm .5$

$n = 245.8$

So take sample size ≥ 246

(c) Suppose the entomologist want to be 98% sure that the average development time has been estimated to within $\pm .5$ days. Should she take a larger or a smaller sample size than the one calculated in part b)? Why?

LARGER sample size needed to decrease probability of making error (1.96 above gets replaced by 2.33, making n larger)

(d) Suppose the entomologist want to be 95% sure that the average development time has been estimated to within ± 1 day. Should she take a larger or a smaller sample size than the one calculated in part b)? Why?

SMALLER sample size needed for less accurate measurement (.5 above gets replaced by 1, making n smaller)

4. A grain dispenser can be set to fill containers of different sizes. If it is set to dispense μ kilos, then the amount it dispenses is a normal random variable with mean μ and standard deviation 20.

(a) Suppose μ is set to 970, and the dispenser is used to fill containers that hold 1000 kilos. What is the probability that a container will be filled to overflowing?

X = amount dispersed = Normal, mean 970
std dev 20

$$P(\text{overflow}) = P(X \geq 1000)$$
$$= P\left(\frac{X-970}{20} \geq \frac{1000-970}{20}\right)$$
$$= P(Z \geq 1.5)$$
$$= .0668$$

(b) What should μ be set to, to make sure that the probability of a 1000 kilo container being filled to overflowing is only 2%?

Want $P(X \geq 1000) = .02$

$$P\left(\frac{X-M}{20} \geq \frac{1000-M}{20}\right) = .02$$
$$P\left(Z \geq \frac{1000-M}{20}\right) = .02$$

Since $P(Z \geq +2.05) = .02$,

Want $\frac{1000-M}{20} = 2.05$, $M = 959$

5. A manufacturer of gunpowder has developed a new powder, which was tested in eight shells. The resulting muzzle velocities, in feet per second, were as follows:

3005	2990	3010	2990
3000	2995	2980	2990

The average of these eight numbers is 2995.

(a) Find s, the sample standard deviation of the sample.

$$s = \sqrt{\frac{(3005-2995)^2+(2990-2995)^2+(3010-2995)^2+(2980-2995)^2 + (3000-2995)^2+(2995-2995)^2+(2980-2995)^2+(2990-2995)^2}{7}}$$

$$= 9.636\ldots$$

(b) Assuming that the muzzle velocities are close enough to normally distributed that the Central Limit Theorem applies for a sample of size 8, find a 95% confidence interval for the true average velocity μ for shells of this type.

$$\left(2995 - \frac{9.636}{\sqrt{8}}\times 1.96,\ 2995 + \frac{9.636}{\sqrt{8}}\times 1.96\right)$$

$$= (2988.3,\ 3001.7).$$

6. Two processes are used in a certain plant to produce the circuit boards for flash drives. To estimate the difference between the proportion of defective boards produced by the two processes, a researcher gathers data as follows, based on random samples:

	Number sampled	Number defective
Process A	80	8
Process B	120	6

(a) Give a point estimate for (proportion defective in process A)-(proportion defective in process B).

$$\left.\begin{aligned}\hat{p}_A &= \frac{8}{80} = .1 \\ \hat{p}_B &= \frac{6}{120} = .05\end{aligned}\right\} \hat{p}_A - \hat{p}_B = .05$$

(b) Compute the SE (standard error) for the distribution of the difference between the two sample proportions.

$$SE = \sqrt{\frac{.1 \times .9}{80} + \frac{.05 \times .95}{120}} = .039$$

(c) Compute the 90% confidence margin of error for the difference between the two sample proportions.

$$\pm 1.645\, SE = \pm .0641$$

Some (possibly) useful formulae

- **Drawing samples from a general population**
  - **Sample mean of a sample of size n:**
$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$
  - **Sample standard deviation of sample of size n:**
$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$
  - **SE of $\bar{x}$ (sample of size n):**
$$SE = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}$$
  - **SE of $\bar{x}_1 - \bar{x}_2$ (sample of size n_1 from population 1, n_2 from population 2):**
$$SE = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \approx \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$
- **Drawing samples from a binomial population**
  - **Sample proportion of a sample of size n:**
$$\hat{x} = \frac{\text{Number of successes}}{n}$$
  - **SE of $\hat{p}$ (sample of size n):**
$$SE = \sqrt{\frac{pq}{n}} \approx \sqrt{\frac{\hat{p}\hat{q}}{n}}$$
  - **SE of $\hat{p}_1 - \hat{p}_2$ (sample of size n_1 from population 1, n_2 from population 2):**
$$SE = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \approx \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$
- **Central Limit Theorem**
  - **Version 1:** If $x_1, \ldots, x_n$ is a random sample from a population with mean μ and standard deviation σ, then for large enough n the distribution of $\bar{x}$ is approximately normal with mean μ, standard deviation $\sigma/\sqrt{n}$.
  - **Version 1:** If $x_1, \ldots, x_n$ is a random sample from a population with mean μ and standard deviation σ, then for large enough n the distribution of $\sum_{i=1}^{n} x_i$ is approximately normal with mean $n\mu$, standard deviation $\sqrt{n}\sigma$.