

MATH 20340, FIRST MIDTERM SOLUTIONS  
FALL 2009

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Q1) 1) 3 simple events in  $A$ ,  $P(A) = \frac{1}{3}$  ( $\frac{3}{9}$ )  
6 " " "  $B$ ,  $P(B) = \frac{2}{3}$  ( $\frac{6}{9}$ )  
3 " " "  $C$ ,  $P(C) = \frac{1}{3}$  ( $\frac{3}{9}$ )

2)  $P(A \cap B) = 0$ ,  $P(A)P(B) = \frac{2}{9}$ , so NO

3)  $P(A \cap C) = \frac{1}{9}$  (just one simple event in common)

$P(A)P(C) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ , so YES

4)  $A$  and  $B$

Q2)  $X$  is Binomial,  $n=12$ ,  $p = \frac{200}{1000} = .2$

1)  $1, 2, \dots, 12$

2)  $P(2 \leq X \leq 5) = P(X \leq 5) - P(X \leq 1)$   
 $= .981 - .275 = .706$

3)  $\mu = np = 12 \times .2 = 2.4$

$\sigma^2 = npq = 12 \times .2 \times .8 = 1.92$

Q3): 1)  $5! = 120$

2) 3 ways to choose winner (one of C, D, E)

2 ways to choose second

3 ways to choose third (from among A, B and whoever of C, D, E wasn't in top 2)

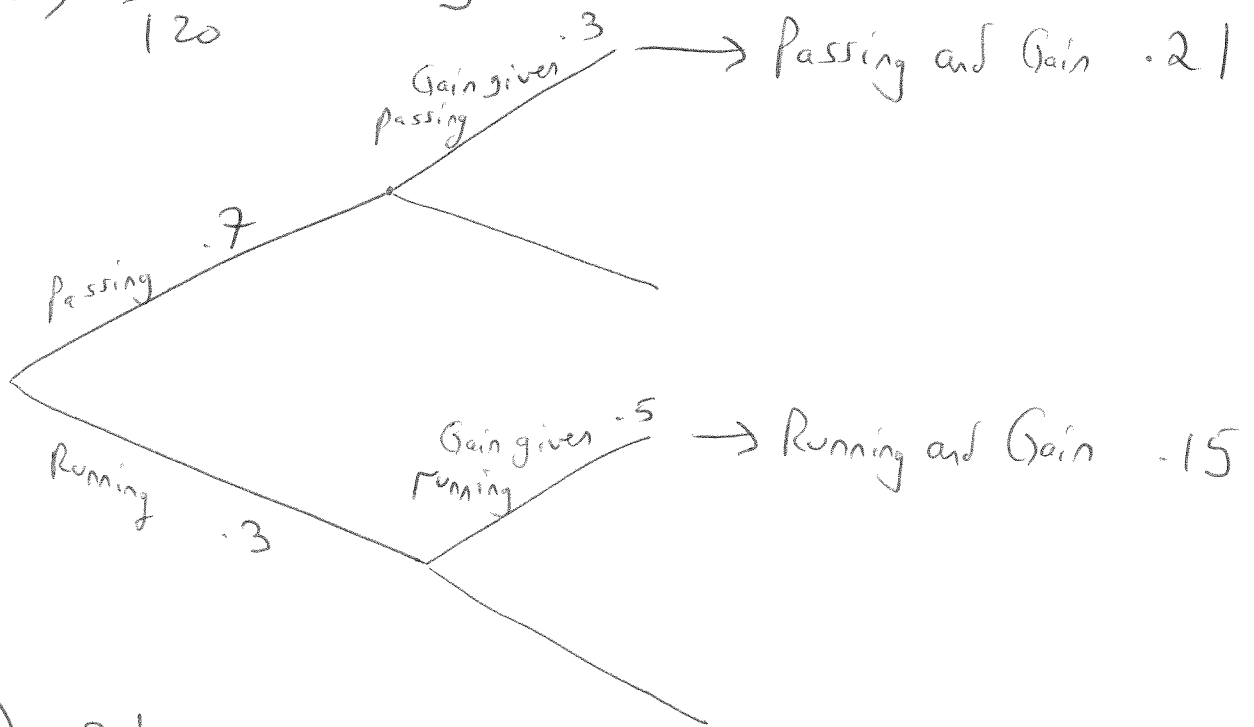
2 ways to choose fourth

1 way to choose fifth

So  $3 \times 2 \times 3 \times 2 \times 1 = 36$

3)  $\frac{36}{120} = .3$

Q4):



1) .21

2)  $.21 + .15 = .36$

3)  $\frac{.21}{.21 + .15} = .583...$   $\left( P(\text{Passing} | \text{Gain}) = \frac{P(\text{Passing and Gain})}{P(\text{Gain})} \right)$

Q5)

$x$	0	1	2	3	4
$p(x)$	.1	-.2	-.4	.2	-.1

$$1) \mu = 0 \times .1 + 1 \times .2 + 2 \times .4 + 3 \times .2 + 4 \times .1$$

$$= 2$$

$$2) \sigma^2 = (0-2)^2 \times .1 + (1-2)^2 \times .2 + (2-2)^2 \times .4$$

$$+ (3-2)^2 \times .2 + (4-2)^2 \times .1$$

$$= 1.2$$

$$3) P(x \geq 2.5) = P(x=3) + P(x=4)$$

$$= .2 + .1 = .3$$

Q6) 1)  $C_{12}^{19}$  (order doesn't matter)

2)  $C_6^9$  ways to choose the six women;  
 for each such way, there are  $C_6^{10}$   
 ways to choose the six men, so  
 $C_6^9 \times C_6^{10}$  in total

$$3) \frac{C_6^9 \times C_6^{10}}{C_{12}^{19}}$$

Q7) One occurrence per 50ml = 20 per litre

$$1) \mu = 20$$

$$\begin{aligned} 2) P(15 \leq X \leq 25) &= P(X \leq 25) - P(X \leq 14) \\ &= .888 - .105 \\ &= .783 \end{aligned}$$

$$\begin{aligned} \text{OR } P(15 < X < 25) &= P(X \leq 24) - P(X \leq 15) \\ &= .843 - .157 \\ &= .686 \end{aligned}$$

$$\begin{aligned} 3) P(X=10) &= P(X \leq 10) - P(X \leq 9) \\ &= .011 - .005 = .006 \end{aligned}$$

This is very low, which seems to validate my suspicion.