Statistics for the Life Sciences

Math 20340 Section 01, Fall 2008

Homework 9 Solutions

- **9.30**:
  - **a**: $H_a : p < .3$ (this is what I want to establish evidence to prove); $H_0 : p = .3$ (this is what I have to accept unless evidence suggests otherwise).
  - **b**: Standard Error is $\sqrt{p_0 q_0/n} = \sqrt{.3 * .7/1000} = .01449....$ (Notice that I am using $p_0 = .3$ to compute the standard error, and not $\hat{p}$. The reason for this is that I am computing the standard error on the assumption that H_0 is true, so I don't need to approximate p — I know it exactly.) Since $z_{.05} = 1.645$, we will accept H_0 for any value of $\hat{p}$ above $.3 - 1.645 * .01449... = .276....$ This is the critical value.
  - **c**: Since our observed $\hat{p}$ is .279, which is greater than .276, there is *not* sufficient evidence to accept H_a at 5%.
- **9.34**:
  - **a**: This is tricky. It feels like we should take the geneticist's claim as the *alternative*, but then our null would be of the form "$p \neq p_0$", and we can only do statistics with a null of the form "$p = p_0$". I think we should look at it like this: the geneticist (an expert) is telling us that there is a sound theoretical reason for saying that $p = .75$, and we are interesting in seeing whether our observations provide sufficient evidence to refute the expert opinion. So $H_0 : p = .75$ versus $H_a : p \neq .75$ seems to be the way to go.
  - **b**: Test statistic: $\frac{.58-.75}{\sqrt{.75*.25/100}} = -3.93...$; p-value is $P(z > 3.93 \text{ or } z < -3.93) = 0$. Results significant at 1% level ... enough evidence to reject null in favour of alternative.
- **9.40**: $H_0 : p = .35$ versus $H_a : p \neq .35$; $\hat{p} = .41$, $n = 300$. Test statistic is $\frac{.41-.35}{\sqrt{.35*.65/300}} = 2.17...$; p-value is $P(z > 2.17 \text{ or } z < -2.17) = .03$. Results not significant at 1% level ... not enough evidence to reject null in favour of alternative.
- **9.42**:
  - **a**: $H_0 : p_1 = p_2$ versus $H_a : p_1 \neq p_2$.
  - **b**: Pooled estimator $\hat{p} = \frac{74+81}{140+140} = .553....$ SE is $\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_1}} = \sqrt{\frac{.553*.447}{140} + \frac{.553*.447}{140}} = .0594....$

- **c**: Test statistic: $\frac{\hat{p}_1 - \hat{p}_2}{SE} = -.84...$. A likely observation.
- **d**: p-value: $P(z > .84 \text{ or } z < -.84) = .4$. Accept null at 1%.
- **e**: Will reject null if test statistic greater than 2.57 or less than -2.57. Since our test statistic is $-.84$, we accept null at 1%.

- **9.46**:

  - **a**: $H_0 : p_1 = p_2$ (p_1 is prop. of adults with children who go regularly to the cinema); $H_a : p_1 \neq p_2$. Test statistic is

  $$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_1}}} = \frac{.2795 - .2589}{\sqrt{\frac{.268 * .732}{440} + \frac{.268 * .732}{560}}} = .73.$$

  Not enough evidence to reject null at 1%.

  - **b**: A difference would be of practical importance because it would suggest to advertisers that they should skew their advertising spending to pitch more to one group than the other.

- **9.48**: The numbers involved here are small, so we should be careful to keep running computations to a good few significant figures to avoid bad rounding errors. $H_0 : p_1 = p_2$ (p_1 is prop. of HRT group with dementia); $H_a : p_1 > p_2$. Test statistic is

  $$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_1}}} = \frac{\frac{40}{2266} - \frac{21}{2266}}{\sqrt{\frac{\frac{61}{4532}}{2266} + \frac{\frac{61}{4532}}{2266}}} = 2.45.$$

  p-value is $P(z > 2.45) = .0071$. Enough evidence at 1% level to reject null, accept alternative.

- **10.2**:

  - **a**: 3.055
  - **b**: 1.746
  - **c**: 2.060
  - **d**: -2.998

- **10.4**:

  - **a**: Stem-and-leaf plot suggests that the normal assumption is not unreasonable.
  - **b**: $\bar{x} = 76.65$, $s = 10.03822$.
  - **c**: SE is $\frac{s}{\sqrt{n}} = 2.2446$. With 19 degrees of freedom, $t_{.025} = 2.093$. So 95% confidence interval is

  $$\bar{x} \pm t_{.025} SE = (71.95, 81.35).$$

- **10.5**:

- **a**: $\bar{x} = 7.05$, $s = .499$.
- **b**: 99% one-sided upper confidence bound: $\bar{x} + t_{.01}\frac{s}{\sqrt{n}} = .74955$ ($t_{.01}$ with 9 degrees of freedom is 2.821.)
- **c**: Test statistic: $\frac{\bar{x}-7.5}{s/\sqrt{n}} = -2.849$. Since critical value for rejecting null is $-t_{.01} = -2.821$, we reject null at 1% significance.
- **d**: Yes. In part b) we found that with probability 99%, the mean lies in an interval that does *not* include 7.5; only lower values.

- **10.8**: $\bar{x} = 60.8$, $s = 7.969$. SE is $\frac{s}{\sqrt{n}} = 2.52$. With 9 degrees of freedom, $t_{.025} = 2.262$. So 95% confidence interval (assuming normal distribution of lengths) is

$$\bar{x} \pm t_{.025}SE = (55, 66.5).$$

- **10.10**:
  - **a**: Yes; the data seems to display a mound-shaped distribution centered around 22 and falling off quickly both above and below 22.
  - **b**: $\bar{x} = 21.4375$, $s = 5.898$.
  - **c**: SE is $\frac{s}{\sqrt{n}} = 1.4747$. With 15 degrees of freedom, $t_{.025} = 2.131$. So 95% confidence interval is

$$\bar{x} \pm t_{.025}SE = (18.29, 24.58).$$

- **10.13**:
  - **a**: $H_0 : \mu = 25$; $H_a : \mu < 25$. Test statistic is $\frac{\bar{x}-25}{s/\sqrt{n}} = -4.3$. With 20 degrees of freedom, $-t_{.005} = -2.845$. So there is strong evidence to reject null.
  - **b**: $(23.23, 29.96)$
  - **c**: It seems that there is a significant increase in self-esteem as a result of treatment, which holds up for at least as long as the time until the follow-up.